Probabilistic Databases: Other Topics and Conclusion

Antoine Amarilli

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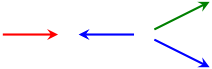
→ We restrict to arity-two signatures (work in progress...)

Homomorphism-closed queries

- A homomorphism from a graph G to a graph G' maps the vertices of G to those of G' while preserving the edges



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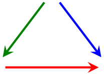

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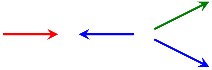
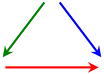
 has a homomorphism to 

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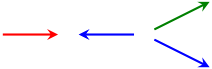
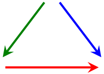
 has a homomorphism to 

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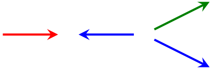
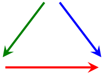
 has a homomorphism to 

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- Homomorphism-closed queries can equivalently be seen as **infinite unions of CQs** (corresponding to their models)

Our result

We show:

Theorem (A., Ceylan, 2020)

For any query Q closed under homomorphisms on an arity-two signature:

- *Either* Q *is equivalent to a tractable UCQ and* $\mathrm{PQE}(Q)$ *is in PTIME*
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We limit to **self-join-free CQs** and extend the "small" Dalvi and Suciu dichotomy to UR:

Theorem (A., Kimelfeld, 2022)

Let Q be a self-join-free CQ:

- *If Q is **hierarchical**, then $\mathrm{PQE}(Q)$ is in PTIME*
- *Otherwise, even $\mathrm{UR}(Q)$ is #P-hard*

Approximate evaluation

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- One possibility is to compute a lower bound and upper bound:
  - $\max(\Pr(\phi), \Pr(\psi)) \leq \Pr(\phi \vee \psi) \leq \min(\Pr(\phi) + \Pr(\psi), 1)$
  - $\max(0, \Pr(\phi) + \Pr(\psi) - 1) \leq \Pr(\phi \wedge \psi) \leq \min(\Pr(\phi), \Pr(\psi))$ (by duality)
  - $\Pr(\neg\phi) = 1 - \Pr(\phi)$ (reminder)

Approximation by sampling

Another possibility is to approximate via Monte-Carlo sampling:

- Pick a random possible world according to the fact probabilities:
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- Approximate the probability of the formula ϕ as the proportion of times when it was true
- Theoretical guarantees: on how many samples suffice so that, with high probability, the estimated probability is almost correct

Other method for a multiplicative approximation: Karp-Luby algorithm

Using external tools

- Specialized software to compute the probability of a formula: weighted model counters
- Examples (ongoing research):
  - c2d: `http://reasoning.cs.ucla.edu/c2d/download.php`
  - d4: `https://www.cril.univ-artois.fr/KC/d4.html`
  - dsharp: `https://bitbucket.org/haz/dsharp`

Repairs

Repairs

- Another kind of uncertainty: we know that the database must satisfy some **constraints** (e.g., functionality)
- The data that we have does **not** satisfy it
- Reason about the ways to **repair** the data, e.g., removing a minimal subset of tuples
- Can we **evaluate queries** on this representation? E.g., is a query true on **every maximal repair**? See, e.g., [Koutris and Wijsen, 2015].

Summary and directions

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- Extensions: homomorphism-closed queries, uniform reliability...

Other topics of research

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Thanks for your attention!

References i

- Abiteboul, S., Kimelfeld, B., Sagiv, Y., and Senellart, P. (2009).
  On the expressiveness of probabilistic XML models.
  VLDB Journal, 18(5).
- Amarilli, A., Bourhis, P., and Senellart, P. (2015).
  Provenance circuits for trees and treelike instances.
  In *ICALP*.
- Amarilli, A., Bourhis, P., and Senellart, P. (2016).
  Tractable lineages on treelike instances: Limits and extensions.
  In *PODS*.

References ii

Amarilli, A. and Ceylan, I. I. (2020).
A dichotomy for homomorphism-closed queries on probabilistic graphs.
In *ICDT*.

Amarilli, A. and Kimelfeld, B. (2022).
Uniform Reliability of Self-Join-Free Conjunctive Queries.
Under review.

Beame, P., Van den Broeck, G., Gribkoff, E., and Suciu, D. (2015).
Symmetric weighted first-order model counting.
In *PODS*.

Benedikt, M., Kharlamov, E., Olteanu, D., and Senellart, P. (2010).
Probabilistic XML via Markov chains.
PVLDB, 3(1).

Berkholz, C. and Merz, M. (2021).
Probabilistic databases under updates: Boolean query evaluation and ranked enumeration.
In *PODS*.

Carmeli, N., Grohe, M., Lindner, P., and Standke, C. (2021).
Tuple-independent representations of infinite probabilistic databases.
In *PODS*.

Ceylan, I. I., Darwiche, A., and Van den Broeck, G. (2021).
Open-world probabilistic databases: Semantics, algorithms, complexity.
Artificial Intelligence, 295.

Cohen, S., Kimelfeld, B., and Sagiv, Y. (2009).
Running tree automata on probabilistic xml.
In *PODS*.

Dalvi, N., Ré, C., and Suciu, D. (2009).
Probabilistic databases: Diamonds in the dirt.
Communications of the ACM, 52(7).

Dalvi, N. N. and Suciu, D. (2004).
Efficient query evaluation on probabilistic databases.
In *VLDB*.

Drien, O., Freiman, M., and Amsterdamer, Y. (2022).
ActivePDB: Active probabilistic databases.
Working draft.

References v

Fink, R. and Olteanu, D. (2016).
Dichotomies for queries with negation in probabilistic databases.
ACM Transactions on Database Systems, 41(1).

Imielinski, T. and Lipski, W. (1984).
Incomplete information in relational databases.
Journal of the ACM, 31(4).

Jung, J. C. and Lutz, C. (2012).
Ontology-based access to probabilistic data with OWL QL.
In *ISWC*.

Kenig, B. and Suciu, D. (2021).
A dichotomy for the generalized model counting problem for unions of conjunctive queries.
In *PODS*.

Koutris, P. and Wijsen, J. (2015).
The data complexity of consistent query answering for self-join-free conjunctive queries under primary key constraints.
In *SIGMOD*.

Olteanu, D. and Huang, J. (2009).
Secondary-storage confidence computation for conjunctive queries with inequalities.
In *SIGMOD*.

Suciu, D. (2020).
Probabilistic databases for all.
In *PODS*.

Widom, J. (2005).
Trio: A system for integrated management of data, accuracy, and lineage.
In *CIDR*.

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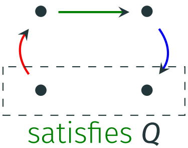

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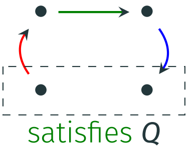

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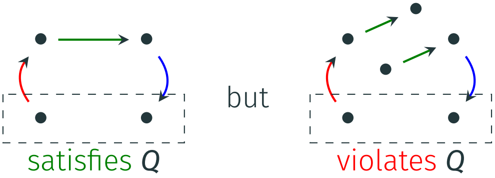

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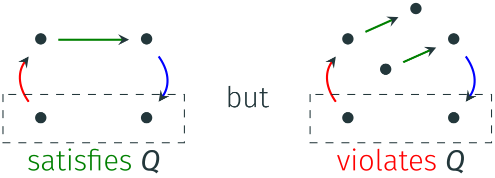

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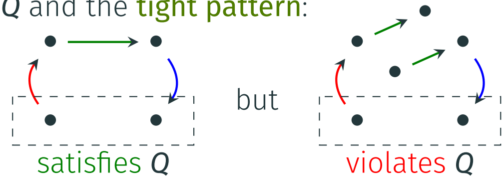

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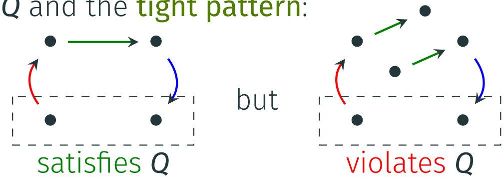

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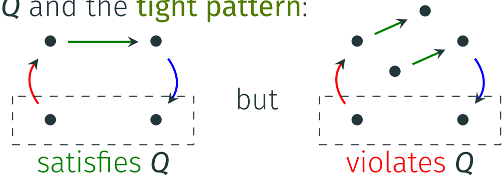

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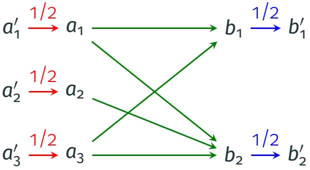

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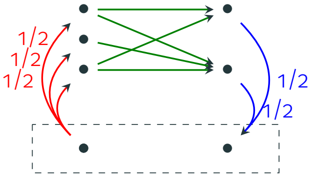

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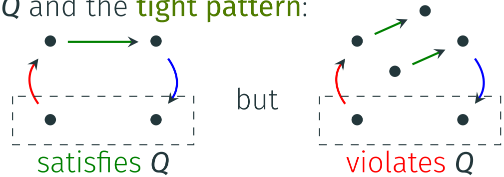

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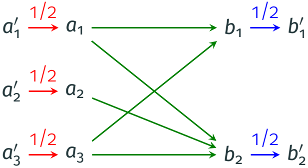

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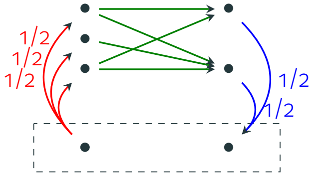

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Using tight patterns to show hardness of PQE

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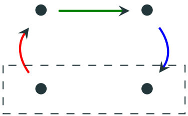

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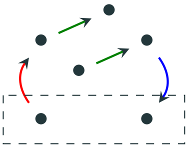

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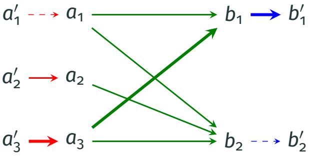

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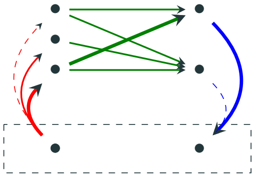

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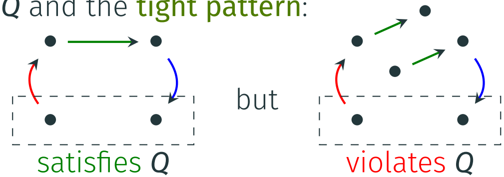

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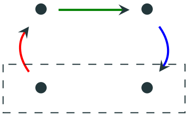

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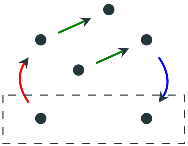

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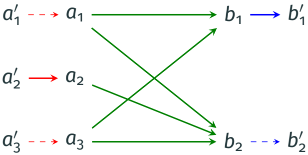

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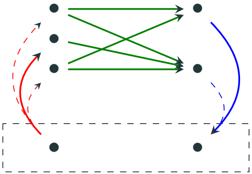

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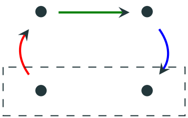

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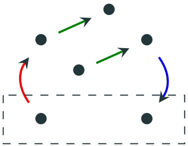

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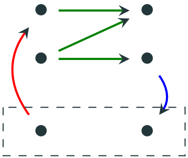

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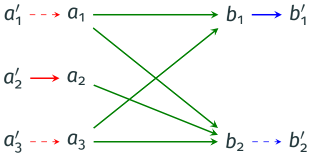

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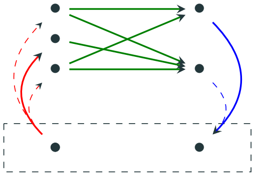

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- Unbounded queries have **arbitrarily large** minimal models
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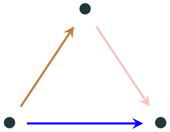

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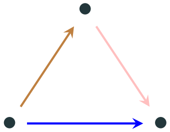

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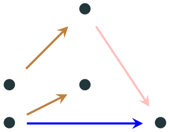

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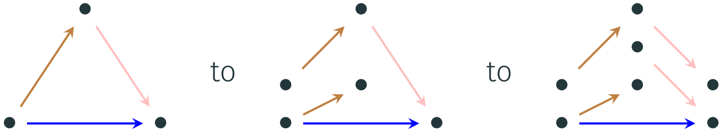

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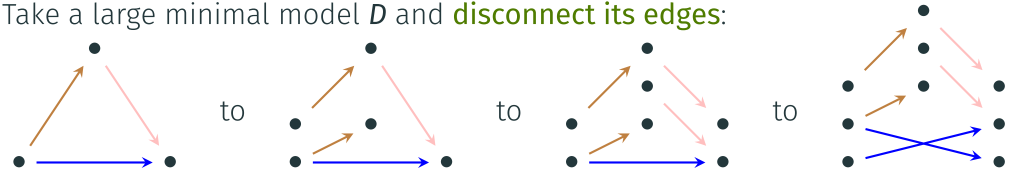

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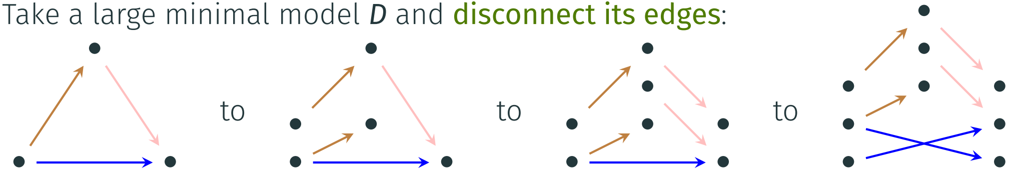

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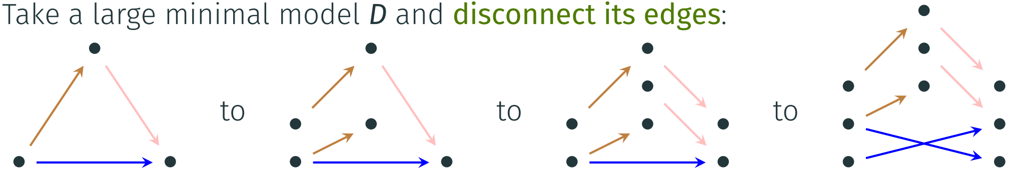

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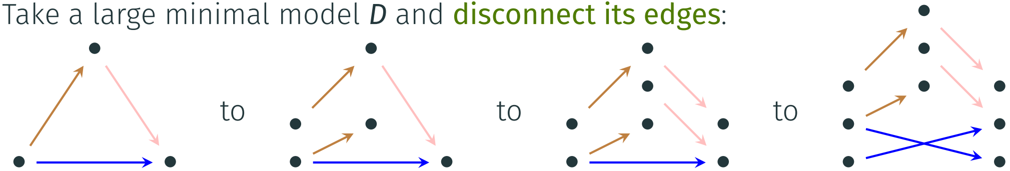

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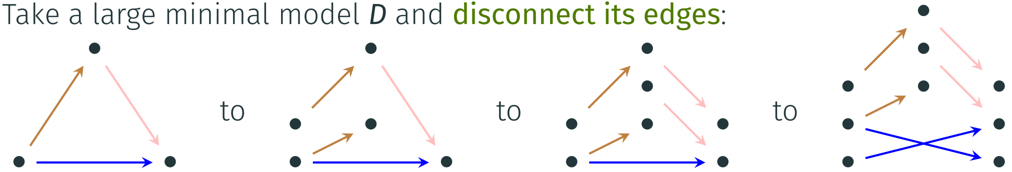

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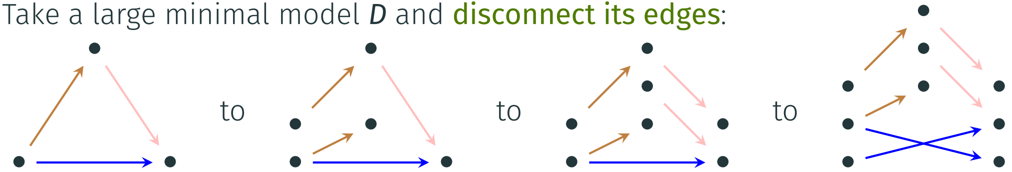

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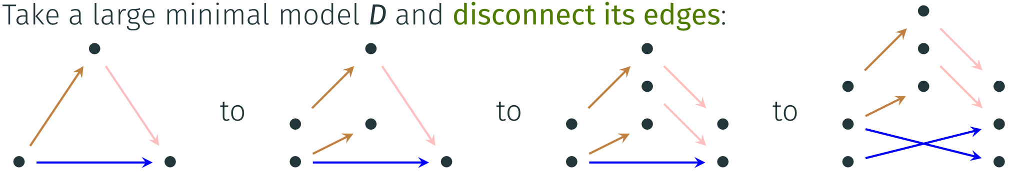

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  - This contradicts the **minimality** of the large D

Rescuing the proof

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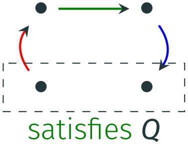

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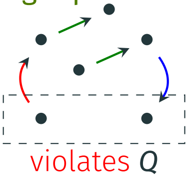

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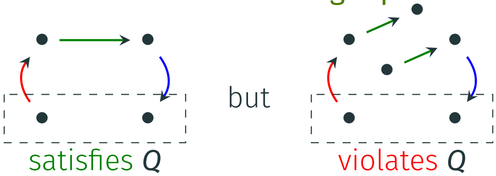

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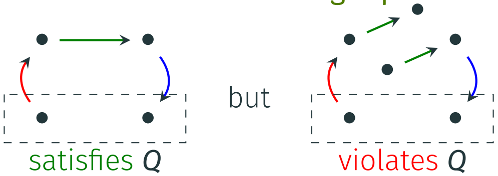

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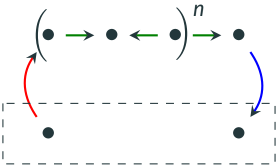

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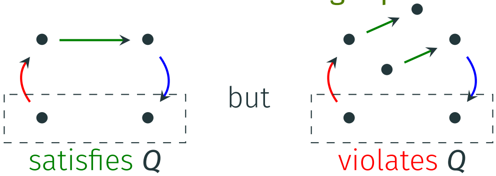

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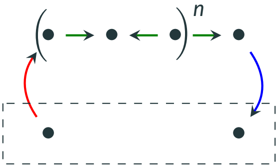

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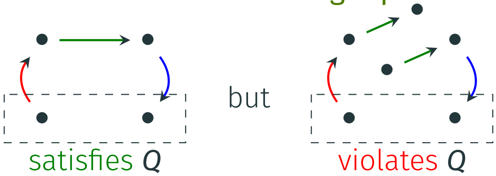

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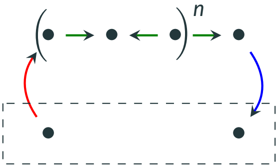

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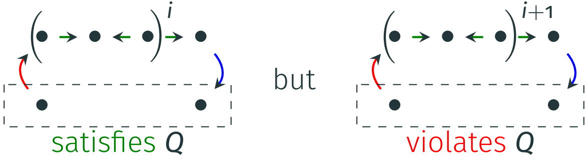

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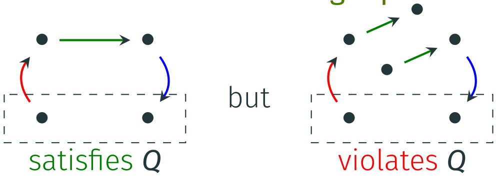

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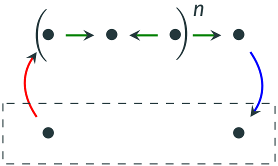

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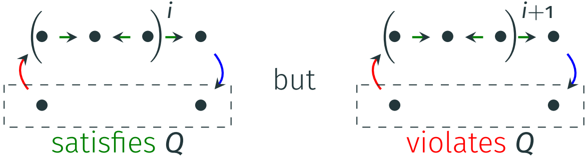

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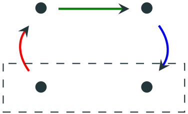

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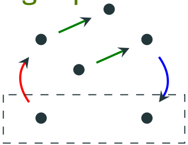

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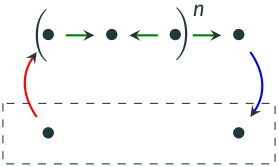

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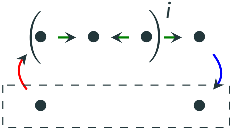

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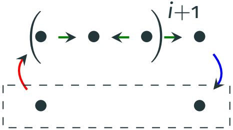

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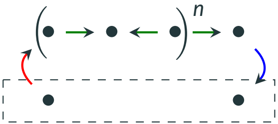

satisfies Q for all $n \in \mathbb{N}$

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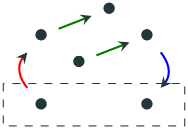

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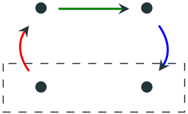

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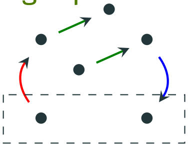

satisfies Q violates Q

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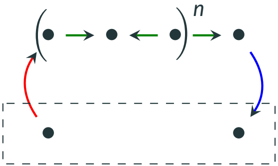

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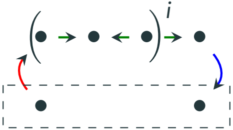

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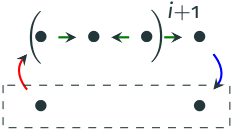

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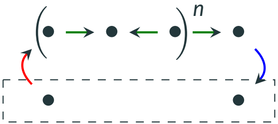

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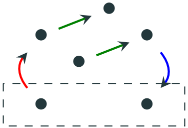

satisfies Q for all $n \in \mathbb{N}$ violates Q

→ Call this an **iterable pattern**

Using iterable patterns to show hardness of PQE

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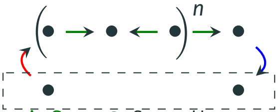

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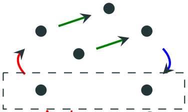

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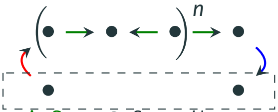

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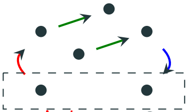

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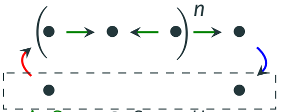

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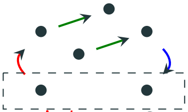

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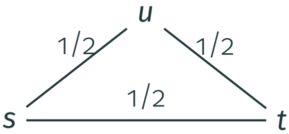

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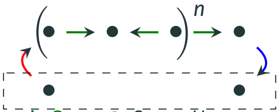

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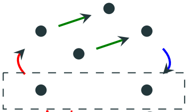

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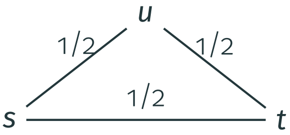

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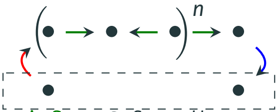

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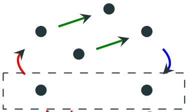

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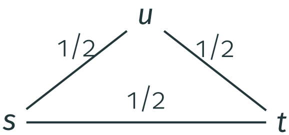

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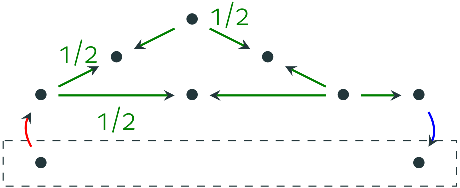

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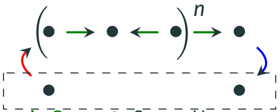

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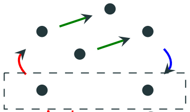

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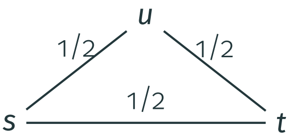

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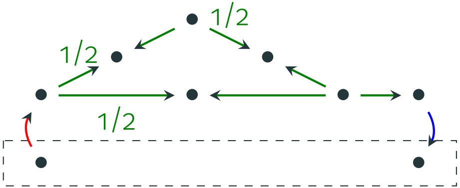

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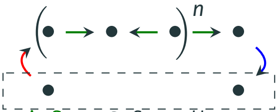

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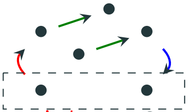

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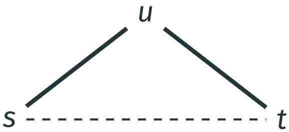

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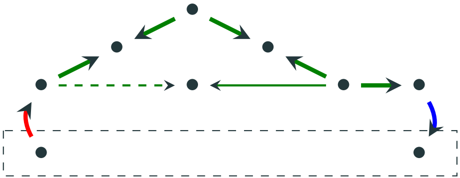

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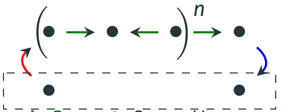

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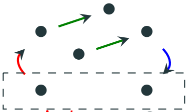

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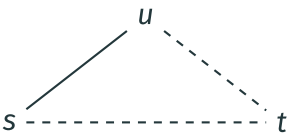

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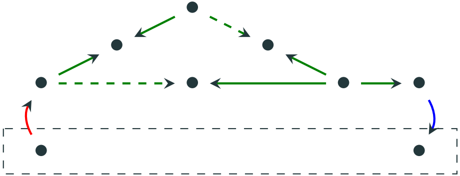

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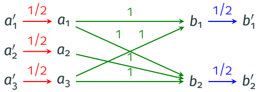

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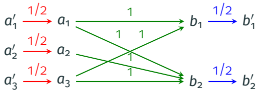

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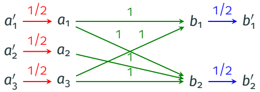

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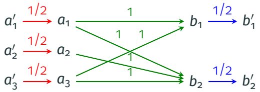

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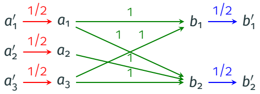

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- Show **invertibility** of this matrix to recover the X_i from the N_i

Using the equation system

We have obtained the system:

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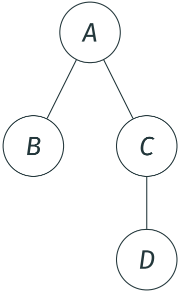
We can choose gadgets and parameters to get a Vandermonde matrix, and show invertibility via several arithmetical tricks

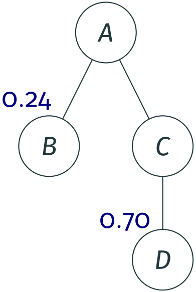
The semistructured model and XML




```
<a>
  <b>...</b>
  <c>
    <d>...</d>
  </c>
</a>
```

- Tree-like structuring of data
- No (or less) schema constraints
- Allow mixing tags (structured data) and text (unstructured content)
- Particularly adapted to tagged or heterogeneous content

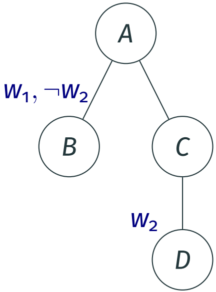
Simple probabilistic annotations



- Probabilities associated to tree nodes
- Express parent/child dependencies
- Impossible to express more complex dependencies
- ⇒ some sets of possible worlds are not expressible this way!

Annotations with event variables

Event	Prob.
w_1	0.8
w_2	0.7



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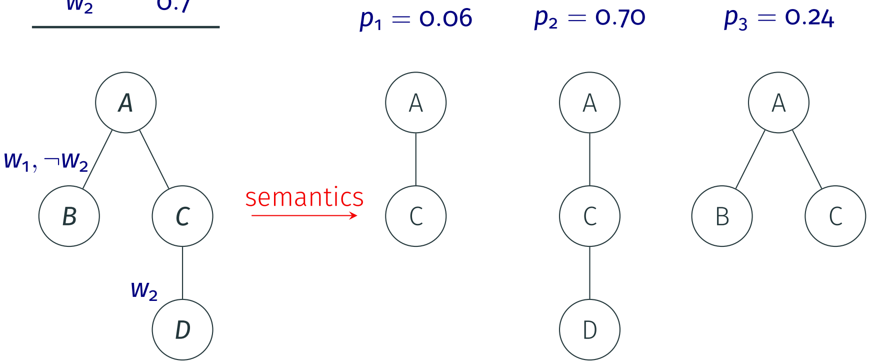

- Expresses arbitrarily complex dependencies

Query evaluation on probabilistic XML

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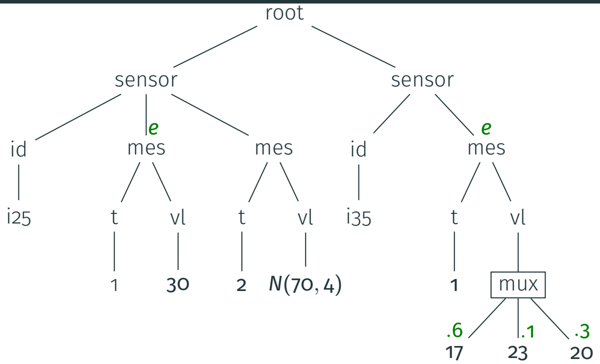
- Bounding the treewidth is **necessary** for tractability in a certain sense [Amarilli et al., 2016]

A general probabilistic XML model [Abiteboul et al., 2009]



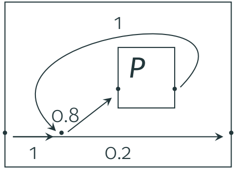
- *e*: event "it did not rain" at time 1
- mux: mutually exclusive options
- $N(70, 4)$: normal distribution

- Compact representation of a **set of possible worlds**
- Two kinds of dependencies: global (*e*) and local (mux)
- Generalizes **all previously proposed models** of the literature

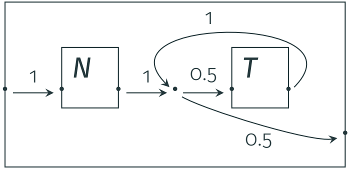
Recursive Markov chains [Benedikt et al., 2010]

```
<!ELEMENT directory (person*)>
<!ELEMENT person (name,phone*)>
```

D: `directory`



P: `person`



- Probabilistic model that **extends** PXML with local dependencies
- Generate documents of **unbounded** width or depth

C-tables [Imielinski and Lipski, 1984]

Patient	Examin. 1	Examin. 2	Diagnosis	Condition
A	23	12	α	
B	10	23	$\bot_1$	
C	2	4	γ	
D	$\bot_2$	15	$\bot_1$	
E	$\bot_3$	17	β	$18 < \bot_3 < \bot_2$

- NULLs are labeled, and can be **reused** inside and across tuples
- **Arbitrary correlations** across tuples
- **Closed** under the relational algebra
- Every set of possible worlds can be represented as a database with c-tables