SD202: Databases

Functional dependencies and normal forms

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Schema normalization

Functional dependencies

Boyce-Codd Normal Form

Conclusion

Schema normalization

Connection to Entity-Relationship

- We know how to design a logical schema via entity-relationship diagrams...
- ... and how to implement it as a physical schema
- The goal of normalization is to check for remaining problems and fix the physical schema
    - Intuitively, we will look for additional constraints in data, called functional dependencies
    - These dependencies mean that tables should be subdivided further

Disclaimer

- The theory of functional dependencies and normal forms is complicated and could fill an entire class!
- We will only see basic insights here

First normal form

A schema satisfies the first normal form if the data of every cell is an atomic type.
For instance, avoid:

Student

id	name	classes
42	John Student	SD202
43	Jane Student	SD202,INF280

→ This should already be the case at the logical schema level, e.g., these attributes should have been composite attributes or multi-valued attributes

Functional dependencies

Definition of a functional dependency

- A functional dependency on a relation R is an assertion of the form $A_1 \ldots A_n \to B_1 \ldots B_m$, where the A_i and B_j are attributes of R
- Semantics: for any two tuples in R, if they agree on all of $A_1 \ldots A_n$ then they agree on all of $B_1 \ldots B_m$

Student

id	name	grade
42	John Student	14
43	Jane Student	16

- The functional dependency **id**, **name** $\to$ **grade** holds

FDs on the data vs FDs on the schema

- An FD is part of the schema: it is a constraint that should always hold
  - $\rightarrow$ "In HotelBookings, the **date** and **room** determine the **reservation_id**"
- The FD will be satisfied on every relation instance of the schema
- However, a relation instance may satisfy some FDs "by chance"

Student

id	name	grade
101	Jean Student	14
102	Jamie Student	14

This data satisfies **name** $\rightarrow$ **grade**, but the schema does not!

FD violations

A violation of an FD $A_1 \dots A_n \to B_1 \dots B_m$ is two tuples that:

- Agree on (all) the attributes $A_1 \dots A_n$
- Disagree on (some of) the attributes $B_1 \dots B_m$

Student

id	name	grade
42	John Student	14
43	Jane Student	14

Example: This demonstrates that the FD **grade** $\to$ **name**, **id**, and the FD **grade** $\to$ **name**, do not hold in the data, hence in the schema

Examples and properties of FDs

- The FD $A_1 \dots A_n \rightarrow B_1 \dots B_m$ always holds if $\{B_1 \dots B_m\} \subseteq \{A_1 \dots A_n\}$
  - $\rightarrow$ For instance, $A \rightarrow A$, or $AA' \rightarrow A$, always hold
  - $\rightarrow$ FDs of this kind are called trivial FDs
- If attributes $A_1 \dots A_n$ are a key for the relation then any FD with (at least) $A_1 \dots A_n$ in the left-hand side will hold
- An FD $A_1 \dots A_n \rightarrow B_1 \dots B_m$ is true iff the FDs $A_1 \dots A_n \rightarrow B_j$ are true for each B_j
  - $\rightarrow$ It suffices to consider the FDs of the form $A_1 \dots A_n \rightarrow B$
  - $\rightarrow$ The general form can still be useful as a shorter notation

Finding FDs

Which FDs hold, and which FDs do not hold, in that instance?

A	B	C	D
1	5	10	4
2	5	11	5
2	5	10	6

Finding FDs

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The FDs $A \to B$, $C \to B$, $AC \to D$, and $D \to ABC$ hold

Finding FDs

Which FDs hold, and which FDs do not hold, in that instance?

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The FDs $A \rightarrow B$, $C \rightarrow B$, $AC \rightarrow D$, and $D \rightarrow ABC$ hold

The FDs $B \rightarrow A$, $B \rightarrow C$, $A \rightarrow C$, $C \rightarrow B$, $A \rightarrow D$, etc., do not hold

FDs and anomalies

- First **find FDs**, by analyzing the business needs
- Some of these FDs are "good", e.g., the ones from the **relation key**
- Others are "bad" and illustrate a **problem in schema modeling**

Student

id	name	supervisor	supervisor_email
42	John Student	Patricia Professor	pprof@telecom-paris.fr
43	Jane Student	Patricia Professor	pprof@telecom-paris.fr
44	Jean Student	Leonard Lecturer	llect@telecom-paris.fr

- Can you find the **bad FD?**

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- Can you find the bad FD? Yes, it is **supervisor_email** $\rightarrow$ **supervisor**

FDs and anomalies

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- Can you find the bad FD? Yes, it is **supervisor_email** $\rightarrow$ **supervisor**
- Can you understand why it will lead to insert/update/delete anomalies?

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Boyce-Codd Normal Form

Boyce-Codd Normal Form

- A set of attributes $A_1 \ldots A_n$ is a **superkey** if it determines the entire relation, i.e., the FDs $A_1 \ldots A_n \to B$ hold for every attribute B
  $\to$ To simplify, we assume **only one key**, then the superkeys are its supersets
- A relation is in **Boyce-Codd Normal Form** (BNCF) if for every **non-trivial** FD $A_1 \ldots A_n \to B_1 \ldots B_m$ that it satisfies, then $A_1 \ldots A_n$ is a **superkey**
- BCNF disallows, for instance:
  - FDs between **non-key attributes** (attributes outside the key)
  - FDs from a **strict subset of the key attributes**

Non-BNCF example (1)

Registration

<u>student</u>	<u>class</u>	teacher
John Doe	SD202	Antoine Amarilli
Jane Doe	SD202	Antoine Amarilli

- This represents a...

Non-BNCF example (1)

Registration

student	class	teacher
John Doe	SD202	Antoine Amarilli
Jane Doe	SD202	Antoine Amarilli

- This represents a... many-to-many relationship between classes and students

Non-BNCF example (1)

Registration

student	class	teacher
John Doe	SD202	Antoine Amarilli
Jane Doe	SD202	Antoine Amarilli

- This represents a... many-to-many relationship between classes and students
- The key is...

Non-BNCF example (1)

Registration

<u>**student**</u>	<u>**class**</u>	**teacher**
John Doe	SD202	Antoine Amarilli
Jane Doe	SD202	Antoine Amarilli

- This represents a... many-to-many relationship between classes and students
- The key is... <u>**student**</u>, <u>**class**</u>

Non-BNCF example (1)

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student	**class**	**teacher**
John Doe	SD202	Antoine Amarilli
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- This represents a... many-to-many relationship between classes and students
- The key is... **student**, **class**
- The **teacher** is...

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- The **teacher** is… an attribute of the relationship

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- The **teacher** is… an attribute of the relationship
- But: the **teacher** in fact…

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- But: the **teacher** in fact... only depends on the class!

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- The key is… **student**, **class**
- The **teacher** is… an attribute of the relationship
- But: the **teacher** in fact… only depends on the class!
- The FD **class** $\rightarrow$ **teacher** holds but…

Non-BNCF example (1)

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- The key is... <u>**student**</u>, <u>**class**</u>
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- The FD **class** $\rightarrow$ **teacher** holds but... **class** is not a superkey (it is a strict subset of the key)

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- Hence, the relation is...

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- The key is... <u>**student**</u>, <u>**class**</u>
- The **teacher** is... an attribute of the relationship
- But: the **teacher** in fact... only depends on the class!
- The FD **class** $\rightarrow$ **teacher** holds but... **class** is not a superkey (it is a strict subset of the key)
- Hence, the relation is... not in BCNF

Non-BNCF example (2)

Candidates

candidate_id	prepa_of_origin	city_of_origin
1	Lycée Kléber	Strasbourg
2	Louis-Le-Grand	Paris

- This table describes the prépa and city of origin of candidates to a competitive exam
- The key is **candidate_id**
- The prépa and city and origin are attributes of the entity
- But: the prépa determines the city!
- The FD **prepa_of_origin** $\rightarrow$ **city_of_origin** holds but **prepa_of_origin** is not a superkey

How to fix BCNF violations? (example)

Take the FD **class** $\rightarrow$ **teacher**, and find all attributes determined by **class**:

student	**class**	**teacher**
John Doe	SD202	Antoine Amarilli
Jane Doe	SD202	Antoine Amarilli

Make two relations:

- One with **class** and with the attributes it determines
- The other with **class** but without the attributes that it determines

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Make two relations:

- One with **class** and with the attributes it determines
- The other with **class** but without the attributes that it determines

student	class
John Doe	SD202
Jane Doe	SD202

class	teacher
SD202	Antoine Amarilli

How to fix BCNF violations? (theory)

- Consider a relation R which is not in BCNF
- Consider a counterexample FD (non-trivial) $A_1 \ldots A_n \to B_1 \ldots B_m$
- Find the closure of $A_1 \ldots A_n$:
  - All attributes B such that $A_1 \ldots A_n \to B$ holds
  - Call this $B'_1 \ldots B'_p$: it contains in particular $B_1 \ldots B_m$

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- Split the attributes between:
  - The FD determiner $A_1 \ldots A_n$
  - The closure $B'_1 \ldots B'_p$ without the FD determiner $A_1 \ldots A_n$
  - The other attributes $C_1 \ldots C_q$

  $\to$ Neither of these sets are empty! (why?)

How to fix BCNF violations? (theory)

- Consider a relation R which is **not in BCNF**
- Consider a **counterexample FD** (non-trivial) $A_1 \ldots A_n \rightarrow B_1 \ldots B_m$
- Find the **closure** of $A_1 \ldots A_n$:
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- **Split** the attributes between:
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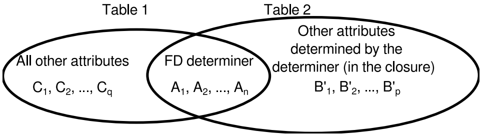
  - The other attributes $C_1 \ldots C_q$

  $\rightarrow$ Neither of these sets are **empty!** (why?)

- **Build** two tables:
  - The projection on $A_1 \ldots A_n$ and $B'_1 \ldots B'_p$
  - The projection on $A_1 \ldots A_n$ and $C_1 \ldots C_q$



Why it works?

- Splitting in two tables **reduces the redundancy**
- Fundamental property: the **join** of the two tables (on the common attributes $A_1 \ldots A_n$) is **equal to the original table**
  - Clearly it contains **at least the same tuples**
  - It cannot contain **more tuples** (why?)

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- Splitting in two tables **reduces the redundancy**
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    - It cannot contain **more tuples** (why?)

  $\rightarrow$ The first table $A_1 \dots A_n, B'_1 \dots B'_p$ **satisfies the FD** $A_1 \dots A_n \rightarrow B'_1 \dots B'_p$

  $\rightarrow$ The rows of the second table will join with **exactly one** row of the first table
- We say that this decomposition is a **lossless decomposition**, as opposed to a **lossy decomposition**

How to compute the closure?

Closure: Given a set of attributes $A_1 \ldots A_n$, how do we compute all attributes B such that the FD $A_1 \ldots A_n \rightarrow B$ holds?

Very simple algorithm:

- Consider all the FDs that you know (when defining the schema)
- Initialize a set $X = \{A_1 \ldots A_n\}$
- Repeatedly go over all FDs until convergence:
  - If an FD $L_1 \ldots L_p \rightarrow R_1 \ldots R_q$ is such that $\{L_1 \ldots L_p\} \subseteq X$
  - Then add $R_1 \ldots R_q$ to X
- At the end, the set X is the closure (why?)

Example of closure computation

Consider the following attributes:

- **item**
- **power**
- **category**
- **color**
- **design_grade**
- **functionality_grade**
- **final_grade**

Consider the FDs:

- **item** is a key
- **category** $\rightarrow$ **color**
- **color** $\rightarrow$ **design_grade**
- **functionality_grade**, **design_grade** $\rightarrow$ **final_grade**

What is the closure of **category**, **functionality_grade**?

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Other topics

- The theory of normalization is **very rich**, we only saw the basics to repair violations in a schema
- Other topics:
  - BCNF is not **dependency preserving**, i.e., sometimes some FDs of the original table are **lost** and cannot be expressed on the BCNF decomposition
  - There is an algorithm to decide which FDs are **implied** by the known FDs (Armstrong's axioms – similar to closure)
  - There are **many other** normal forms!

Credits

Sources:

- https://pierre.senellart.com/enseignement/2016-2017/bd/6-normalisation.pdf
- https://sites.google.com/site/bahrimarouaa/teaching/inf725
  "Functional dependencies and normalization"