Write a solution for each exercise in two or three sentences along with the complexity computation. Unless specified otherwise your program should run in 1s with a reasonable amount of RAM (i.e. 100 Mo).

Exercise 1: Marienbad

You are playing a two-player game with $N \leq 10^5$ marbles. At their turn, the current player needs to remove between 1 and 3 marbles. The player that removes the last marble loses. It is your turn to play, do you have a way to ensure your victory ?

Exercise 2: Marienbad 1.5

Same as Marienbad except that this time you are given an array T and when there are i marbles remaining, the current player can remove between 1 and T_i marbles. You are guaranteed that $T_i \leq 10$ for each i. (Remark : the previous exercise corresponds to the case where we have $T_i = 3$ for each i.)

Exercise 3: Balance

You have a balance with $N \leq 10^3$ weights each weighting between 1 and 10^3 grams. Can you find a subset of the weights weighting a total of $T \leq 10^4$ grams ?

Exercise 4: Path in grid

We have a matrix $L \times R$ of numbers $M_{i,j}$. In that grid, we consider the paths going from the cell $(0, 0)$ to the cell $(R - 1, L - 1)$ that are only going down or right. The weight of a path is the sum of the cells it goes through, what is the maximal weight of a path ?

You know that $1 \leq L, R, M_{i,j} \leq 10^3$.

Exercise 5: Longest path

Given a DAG with $N \leq 10^4$ nodes and $M \leq 10^6$ edges, find the longest path starting from node 0.

Exercise 6: Nice necklace

You want to create your own necklace using $N \leq 10^5$ pearls each having a color among the $K \leq 10^5$ colors available. You can have several pearls of the same color and you even allow two adjacent pearls to be of the same color but it is not possible to have three adjacent pearls of the same color as that would be ugly. What is the number of ways you can create such a necklace ?

The answer might be huge so compute the answer modulo 1000003.

Exercise 7: Marienbad 2

You are playing a two-player game with $R \leq 10^3$ red marbles and $B \leq 10^3$ blue marbles. At their turn, the current player needs to remove between 1 and 3 marbles all of the same color. The player that removes the last marble loses. It is your turn to play, do you have a way to ensure your victory ?

Exercise 8: Trekking

You are going to do trek and you plan on carrying your own food. You already went shopping for groceries and it is now time to pack. You shopped for $N \leq 10^3$ different items, each item i having a weight w_i in grams $0 \leq w_i \leq 10^3$ and a number of $0 \leq c_i \leq 10^3$ calories. What is the minimal weight that you can achieve to have a total of at least $1 \leq C \leq 10^4$ calories ? Each item can be taken only once.