INF280: Competitive programming

Advanced datastructure algorithms

Louis Jachiet

Sliding windows

Sliding window techniques to improve efficiency

Typical examples using a list $i_1, \ldots, i_N$ and an integer K

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Naive algorithm

Two (or three!) nested loops, recomputing from scratch for each position j.

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Sliding window idea

Optimize away nested loops!

Example: fixed width (e.g. maintaining sum of K elements)

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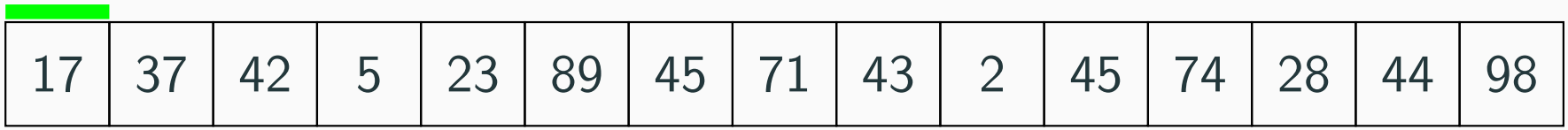

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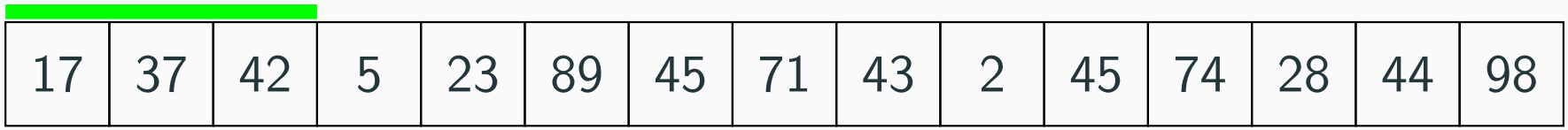

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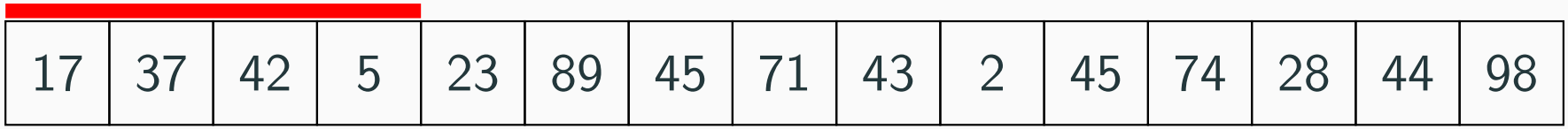

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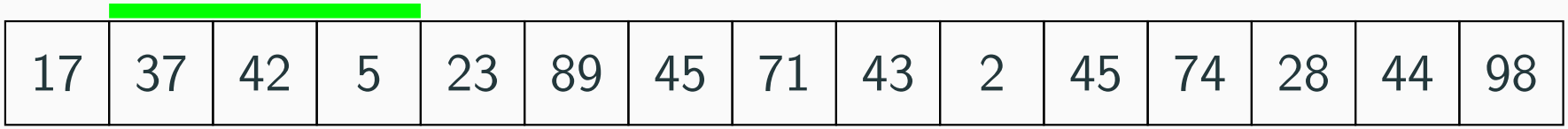

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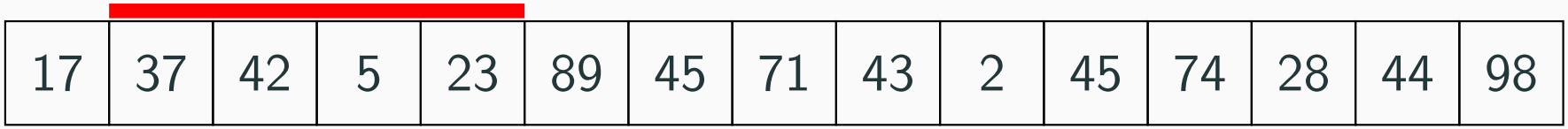

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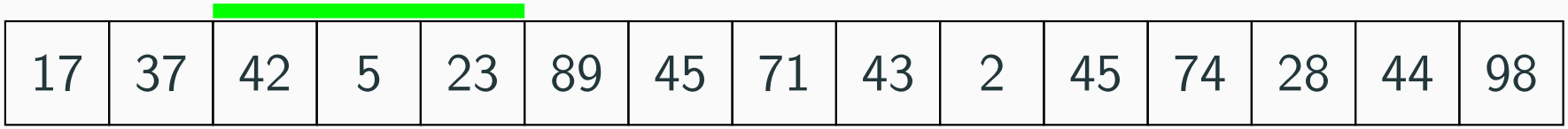

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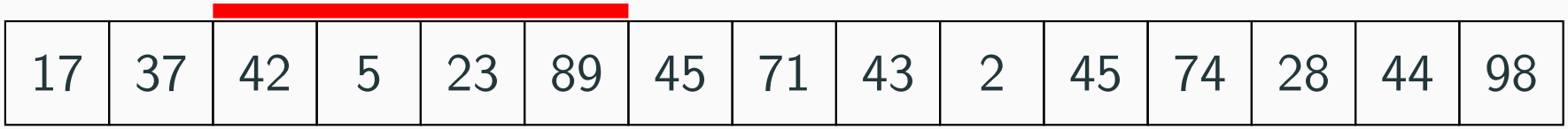

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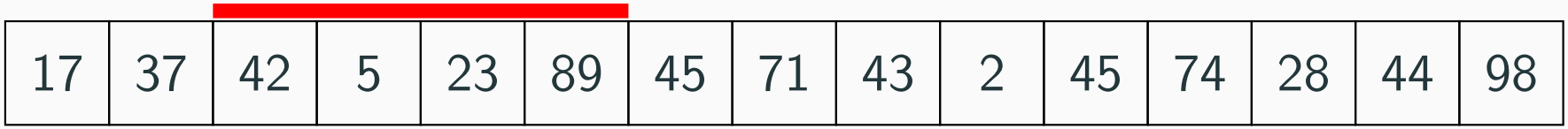

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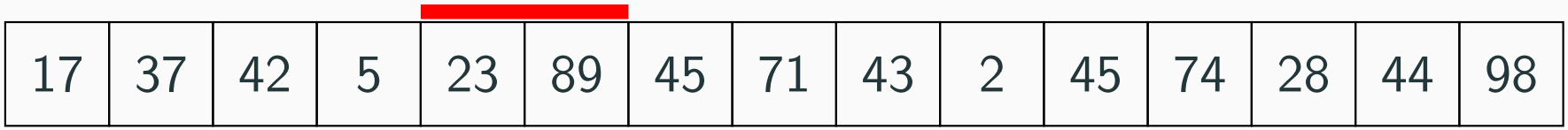

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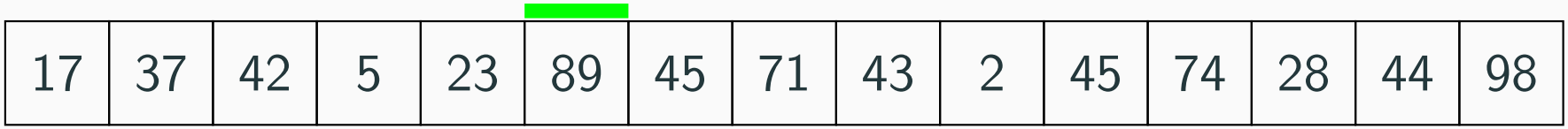

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General idea

Maintain a double ended queue where:

- you add on the right to grow
- remove on the left to shrink
- maintain some computation over the window content

Works with monotone criteria for windows!

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The deque trick to maintain min and max

Maintain the (ordered) list of elements that might become min/max.

Example: maintaining min of 5 elements)

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Updates

- Add on the right: remove everything bigger
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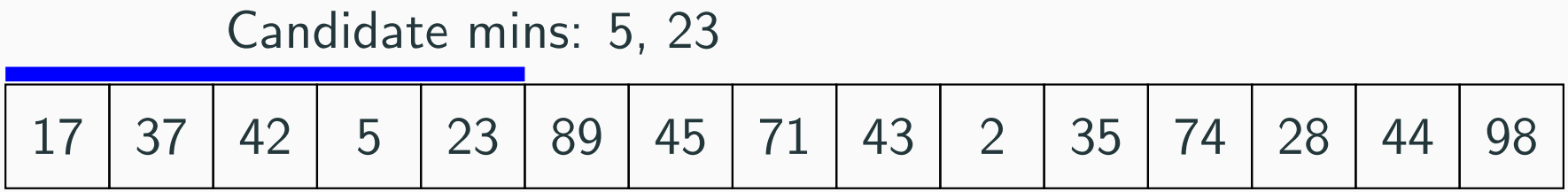

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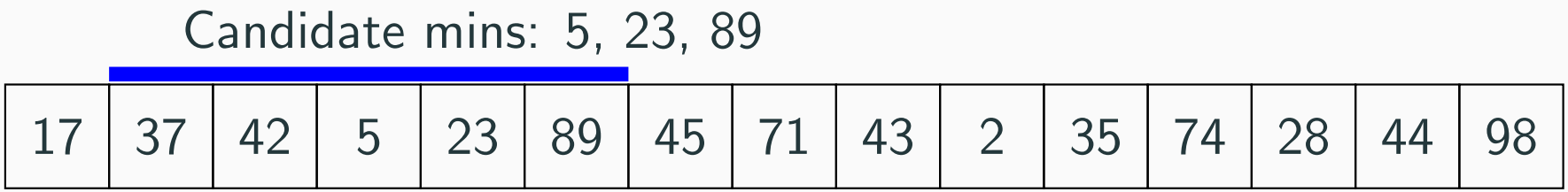

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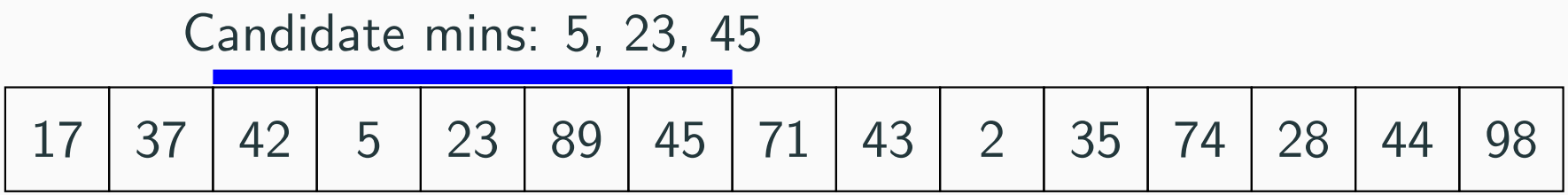

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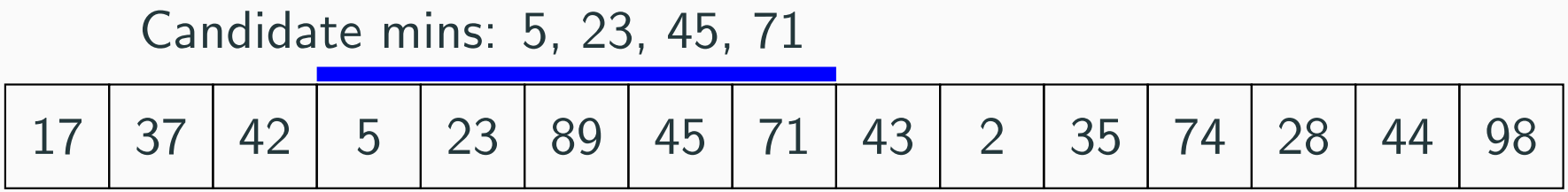

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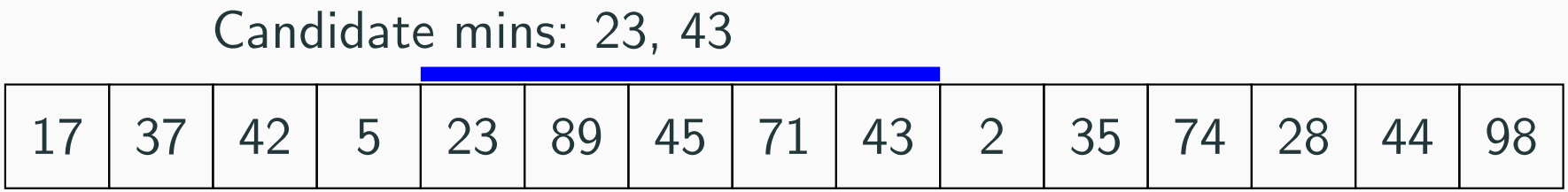

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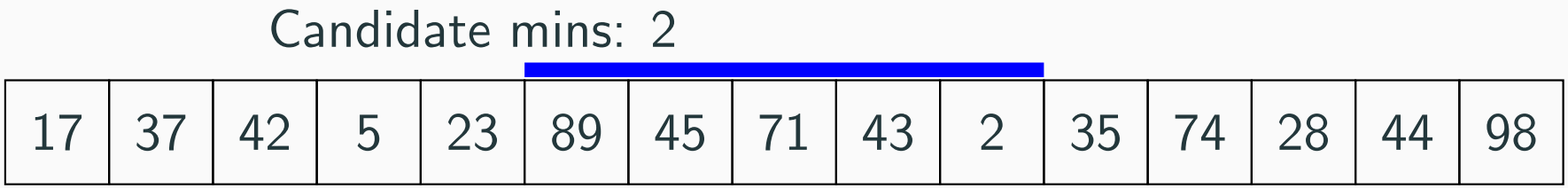

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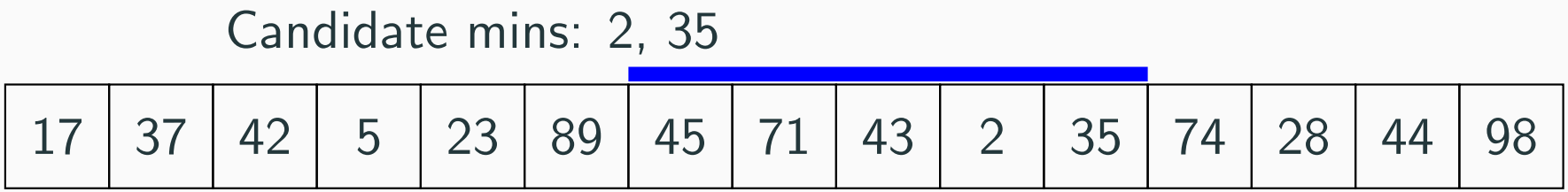

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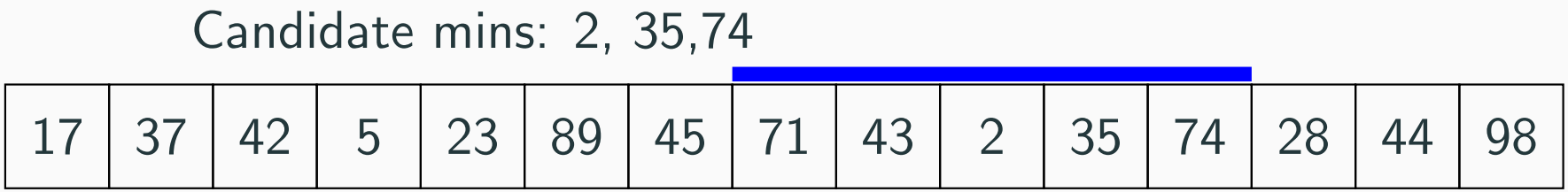

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Also works with a set... with a $O(\log(n))$ penalty.

Prefix sums

Structure to compute sums in $O(1)$

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A list of elements $v_1 ... v_n$ over a group

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Compute $q(i,j) = \sum_{i \leq \ell < j} v_\ell$

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