Graph databases

Graph Databases?

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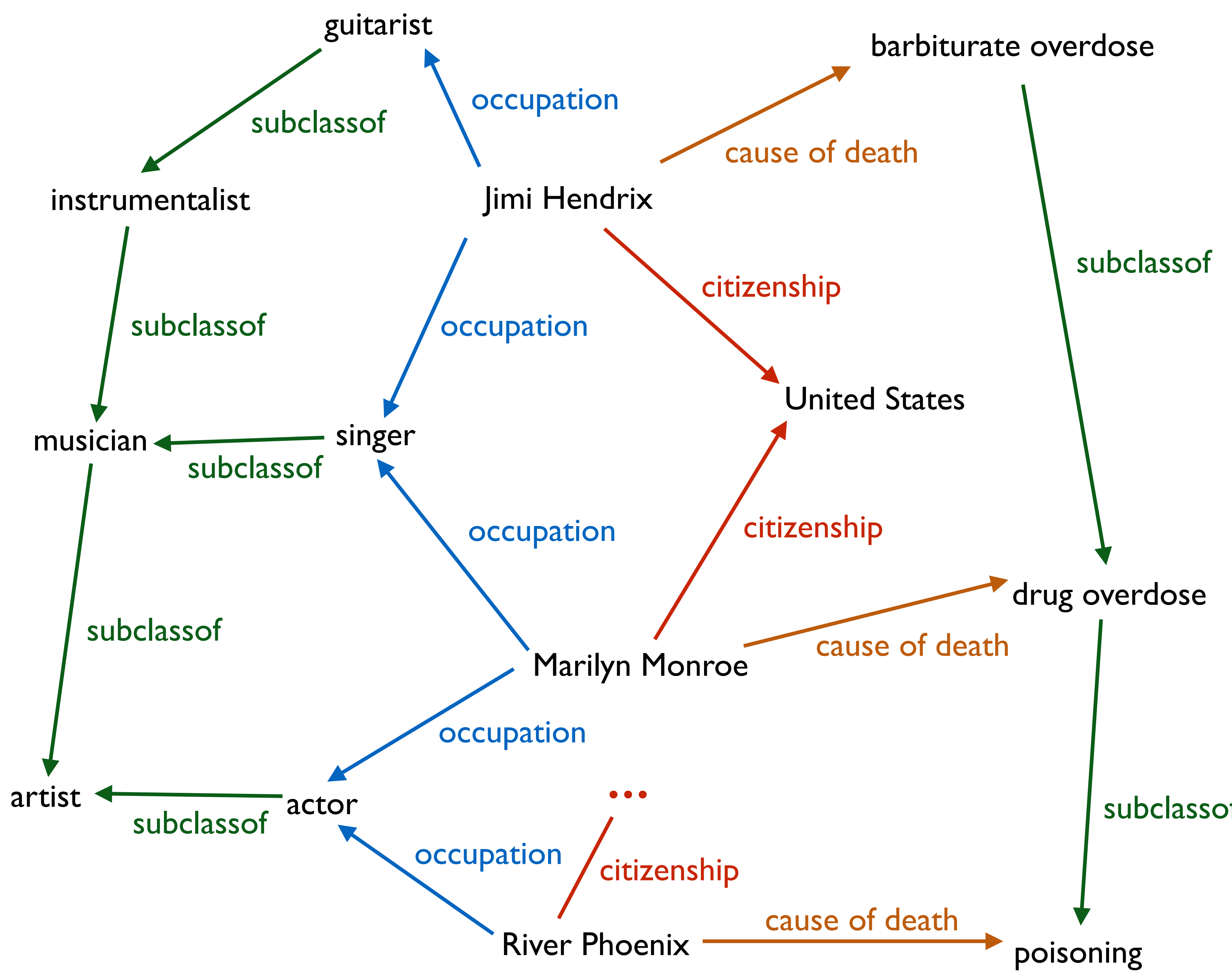

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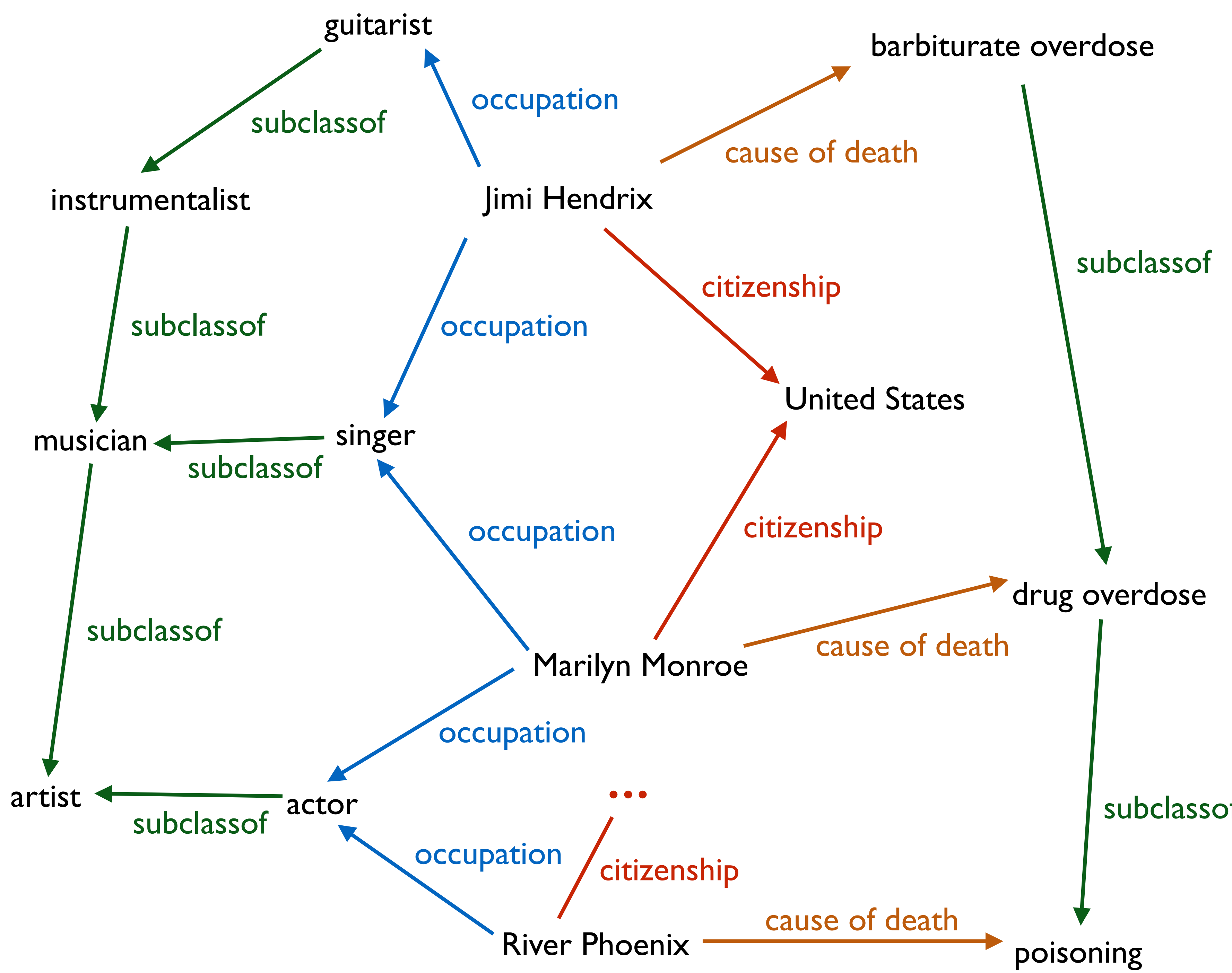

Information stored as a graph

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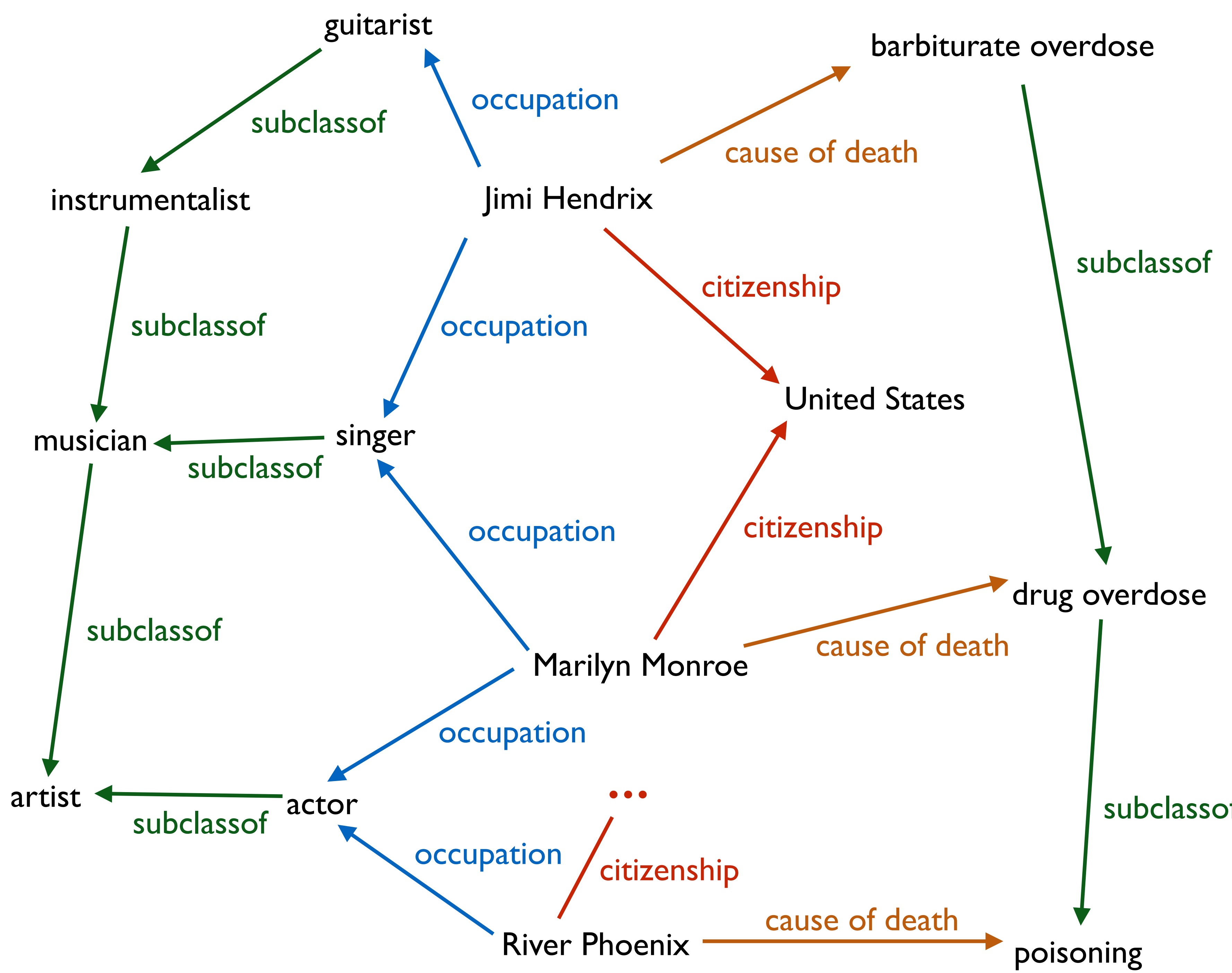
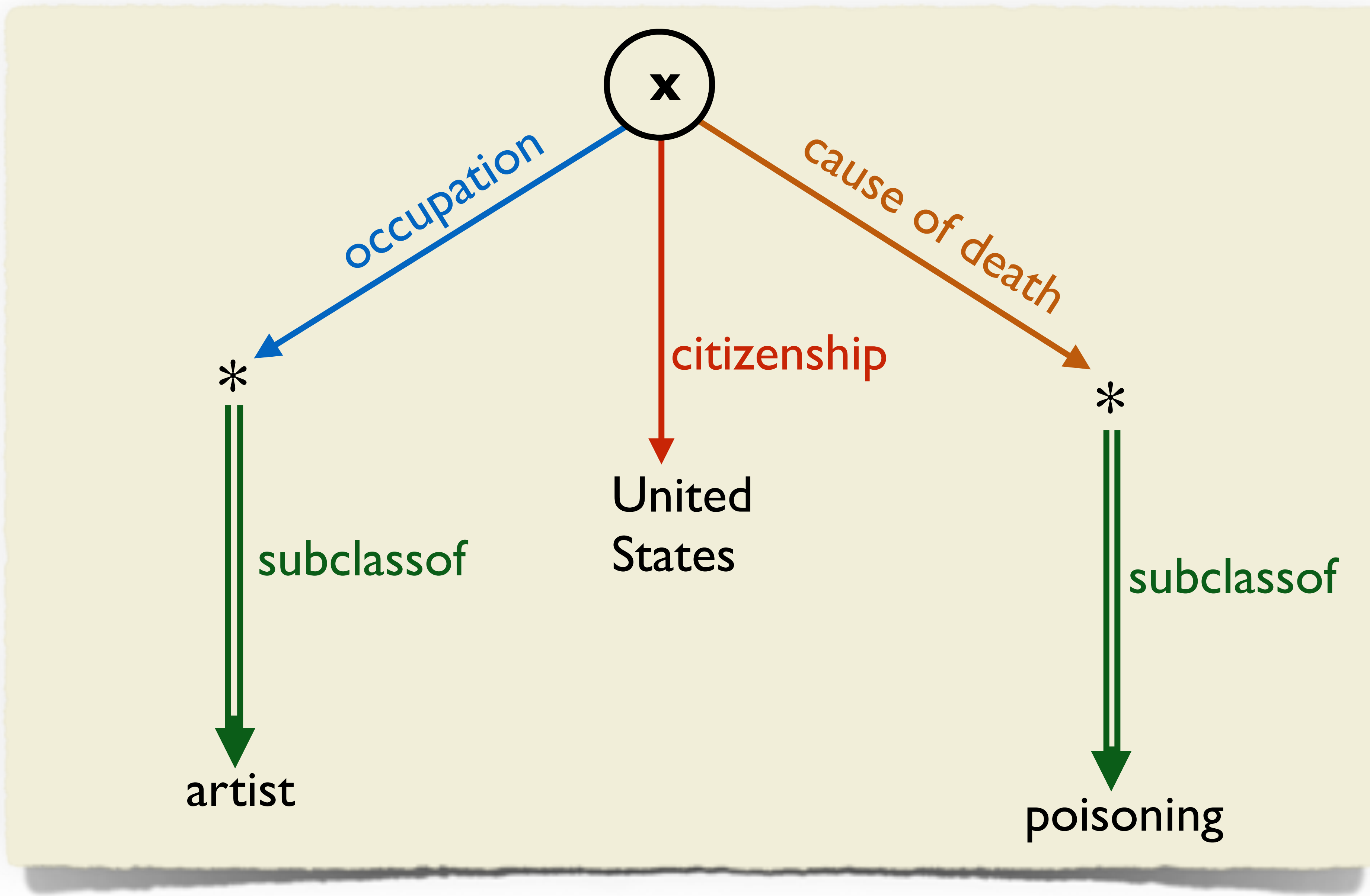


Information stored as a graph

Rather intuitive

The Query, Visualized

"US artists who died of poisoning"



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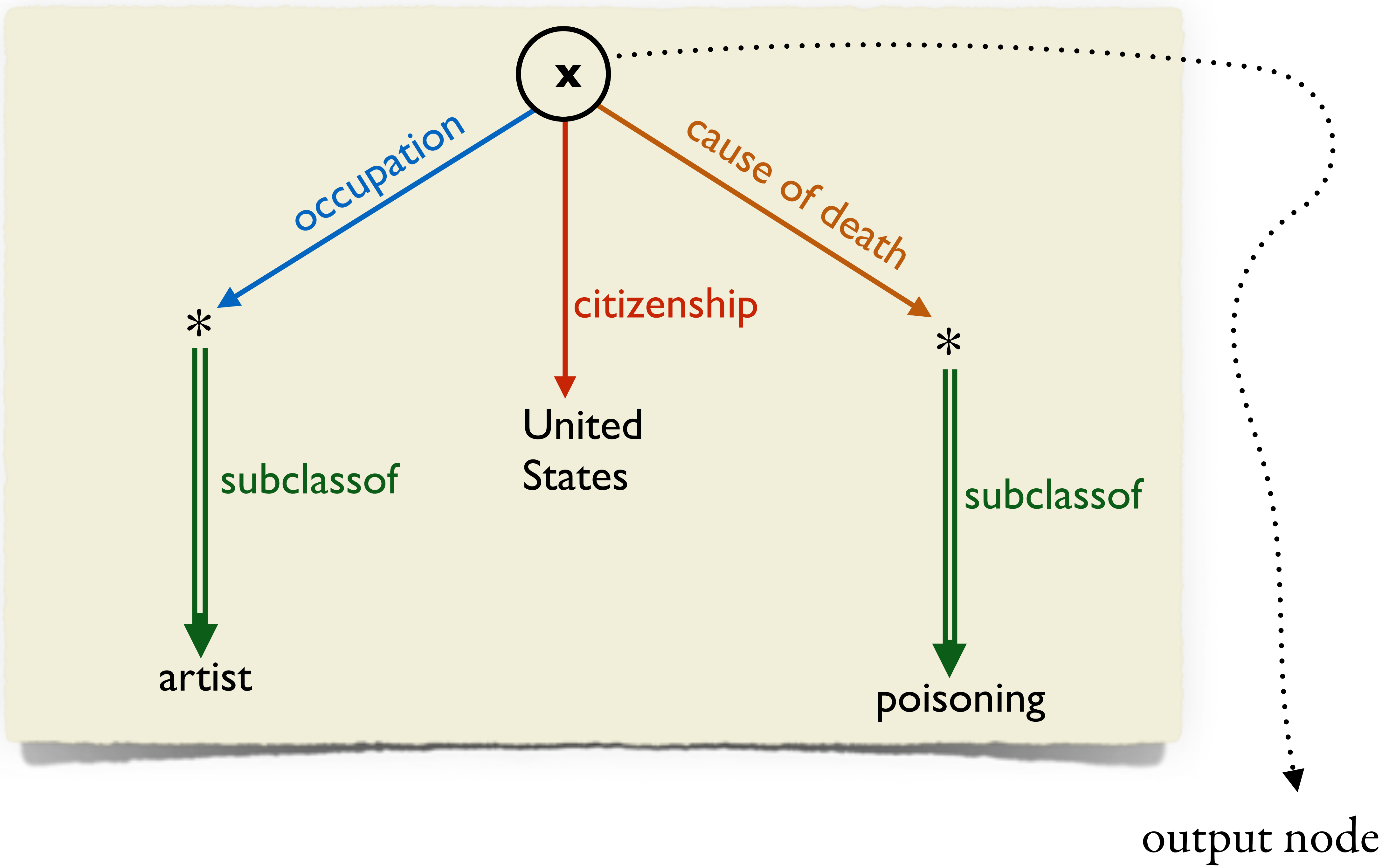

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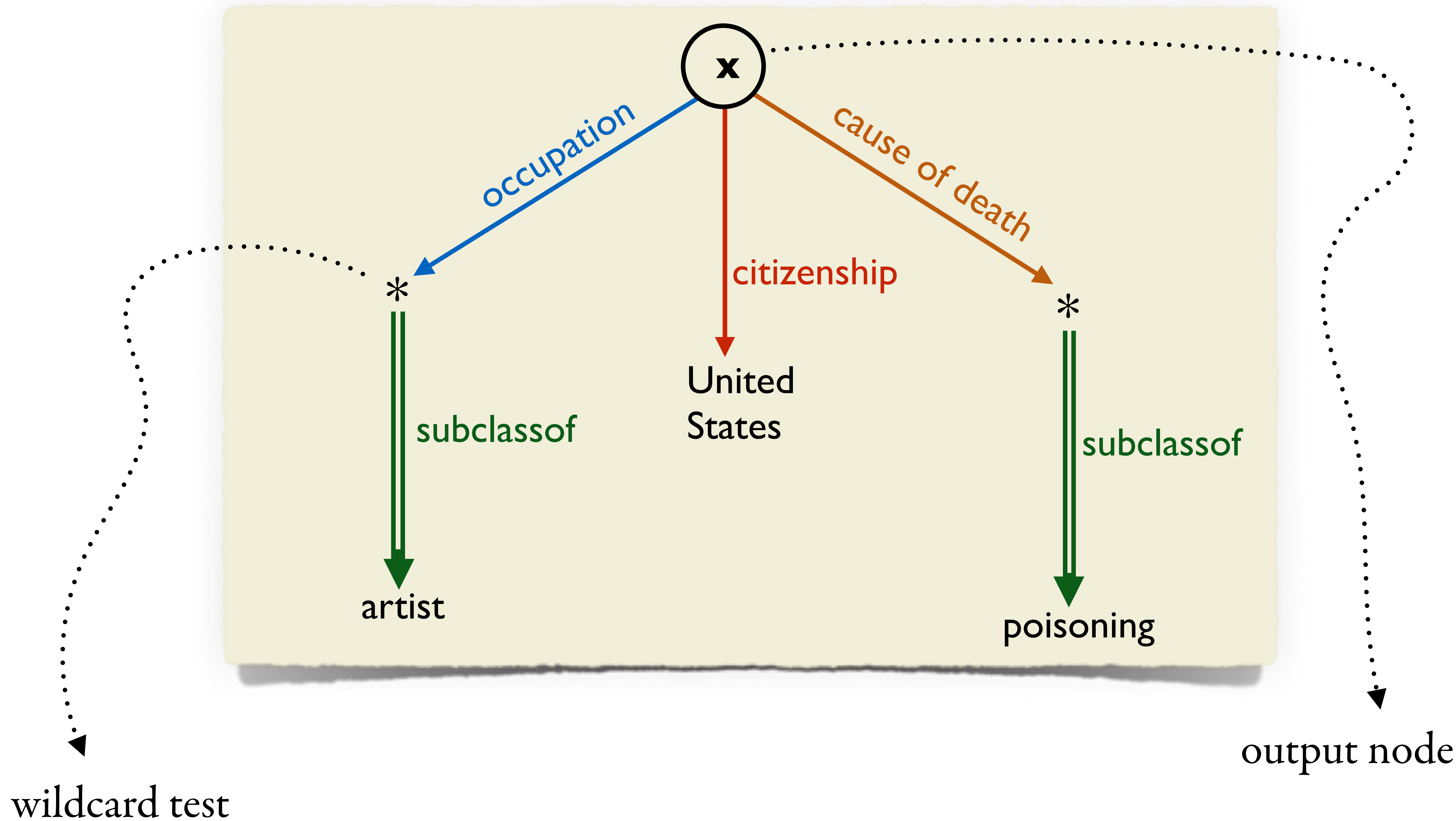

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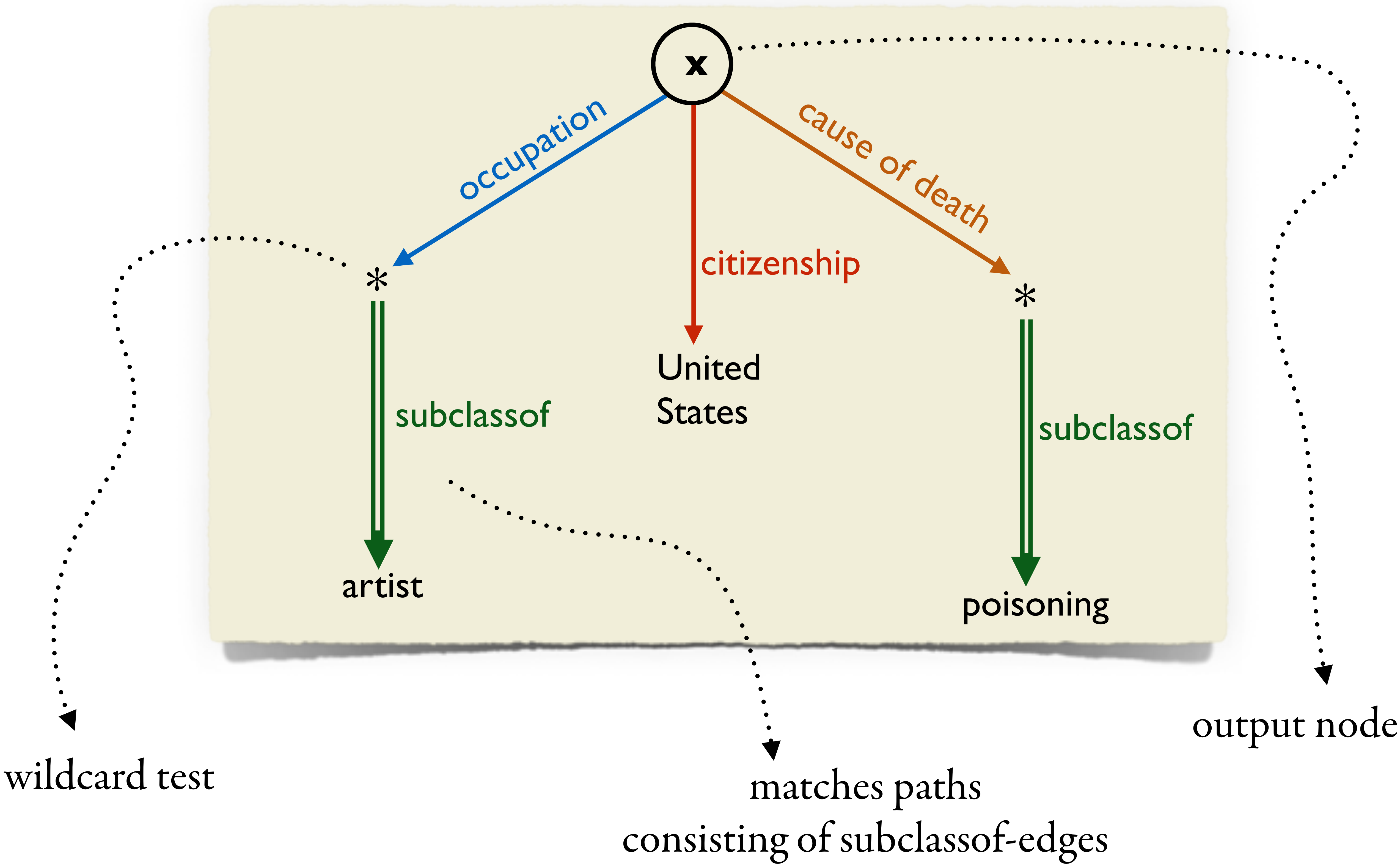

Graph Queries By Example

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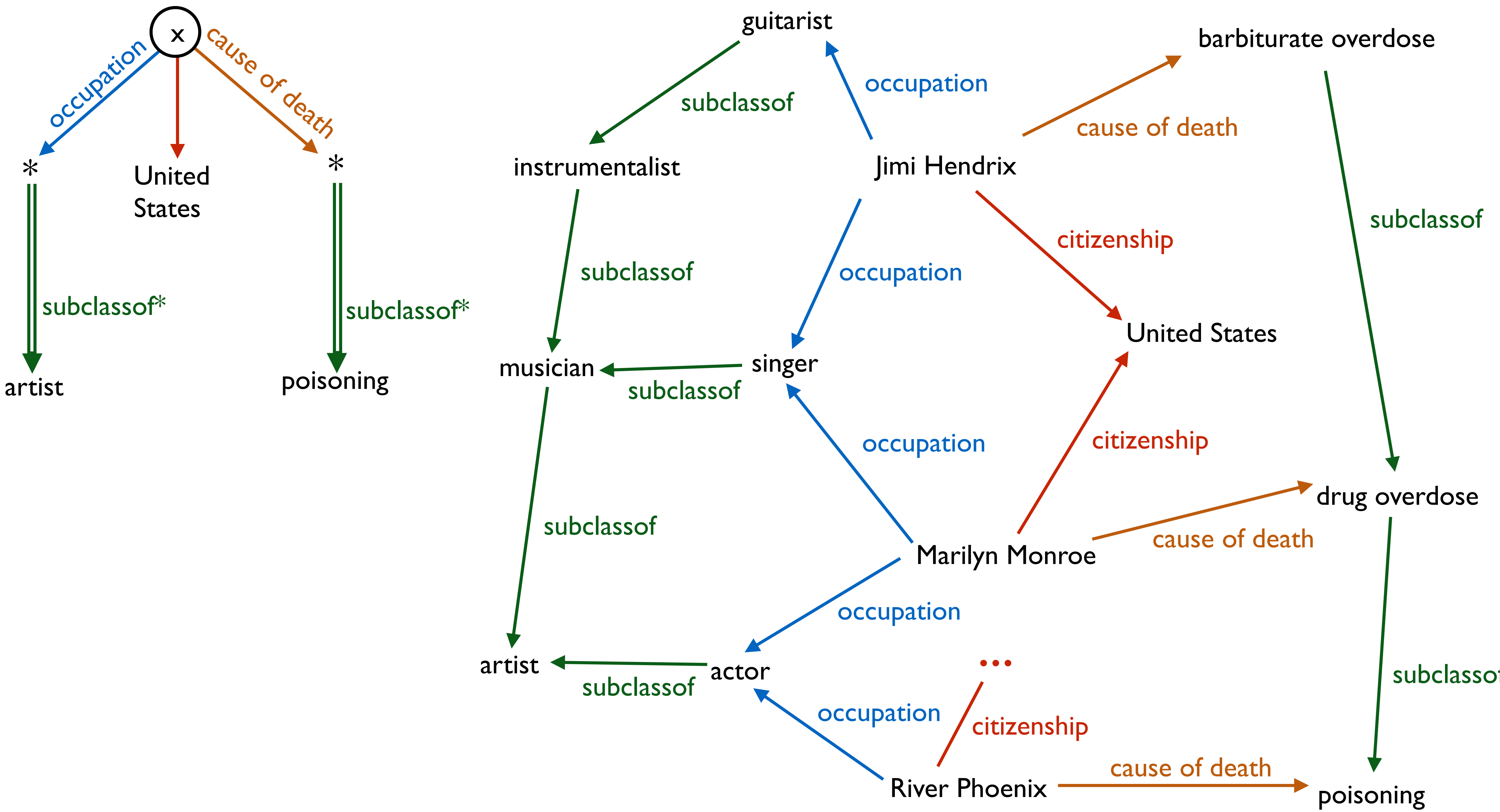

Graph Queries By Example

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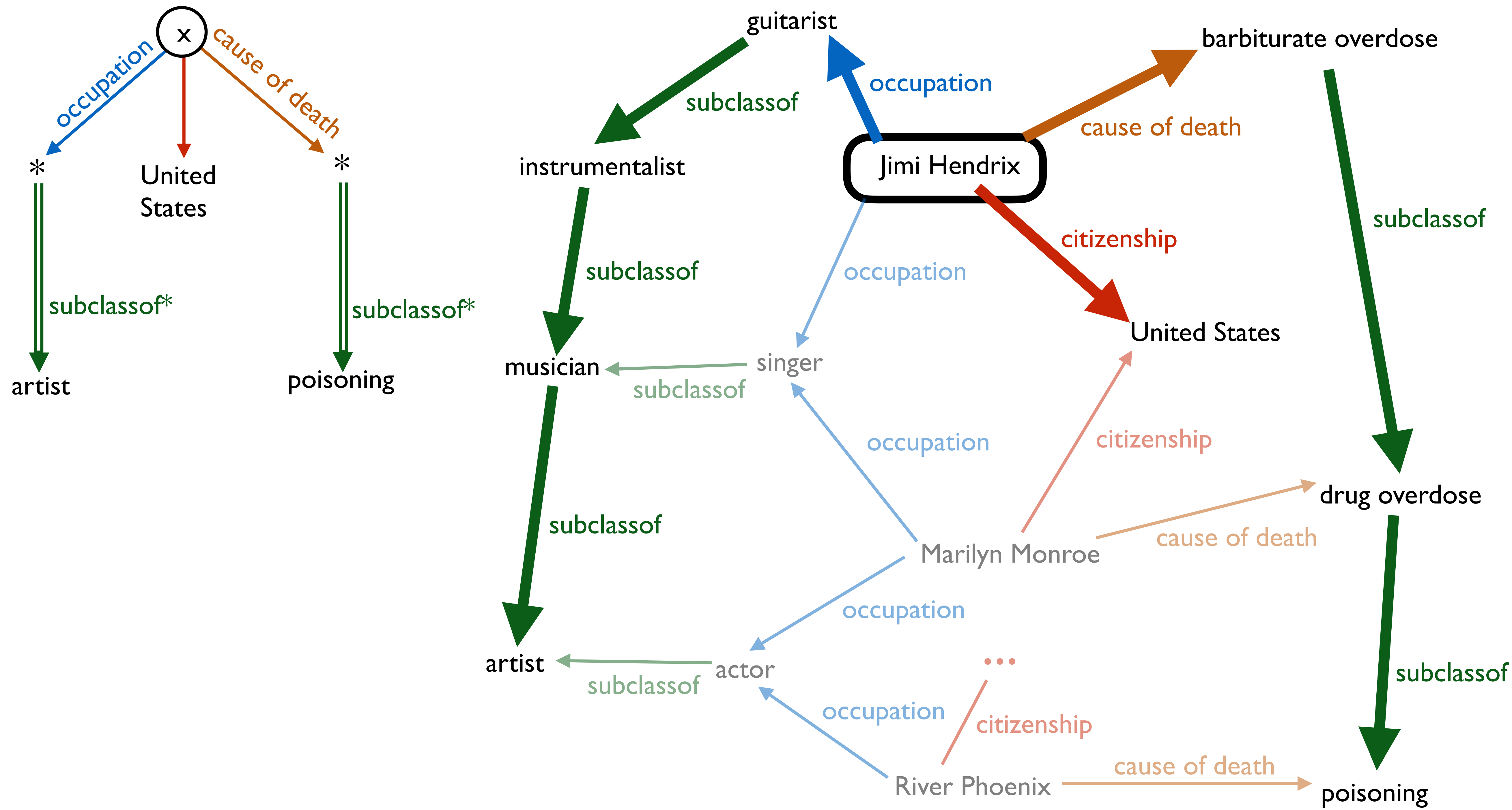
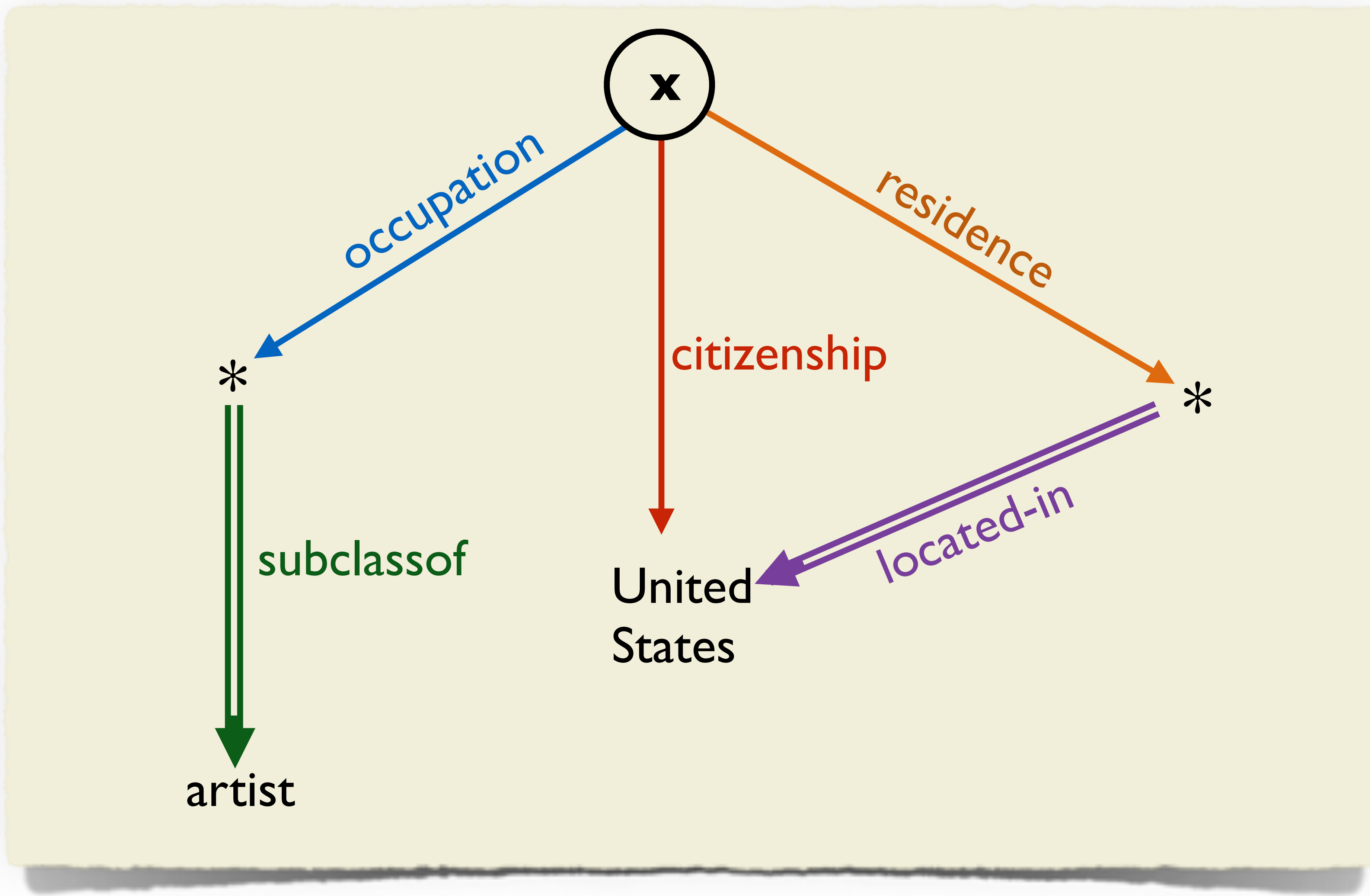


Graph Queries By Example

Queries can have cycles



Artists who live in the US and have US citizenship

Why Graph Databases?

Why are they interesting?

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Graph DBs are becoming standard in Industry

Oracle, Neo4j (about 50% of the market), Tigergraph, Redis, SAP, ArangoDB, Amazon Neptune, etc etc
Often hidden: e.g., Google's Knowledge Graph

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The first one they developed is SQL

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New Applications

Social networks, Semantic Web, bioinformatics, fraud analysis, real-time recommendation, network/IT systems, even investigative journalism (Panama+Pandora papers)

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Future in Analytics

Gartner prediction: in the next 5 years, up to 80% of all analytics task will involve graph databases

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Current and future use

75% of Fortune 100 companies currently use graph databases

Phenomenal fundraising (last year alone, around 500M)

GQL Influence Graph

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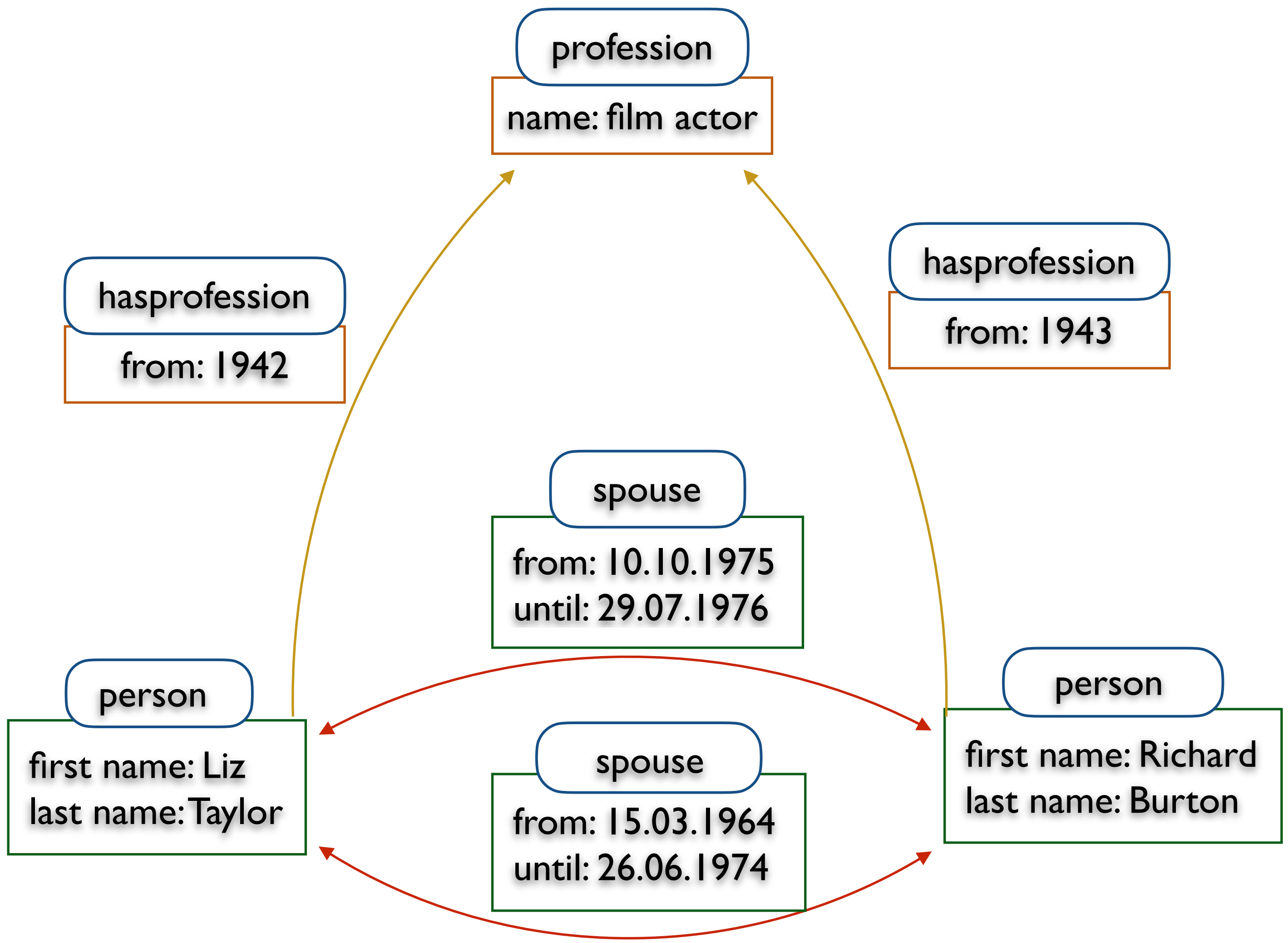
[https://www.gqlstandards.org/existing-languages]

Models for Graph Databases?

Currently, two main data models:

- Property Graph Databases (today: the dominant model)
- RDF-like Databases (an earlier and interesting approach but not as prevalent in industry)

Property Graph Data Model



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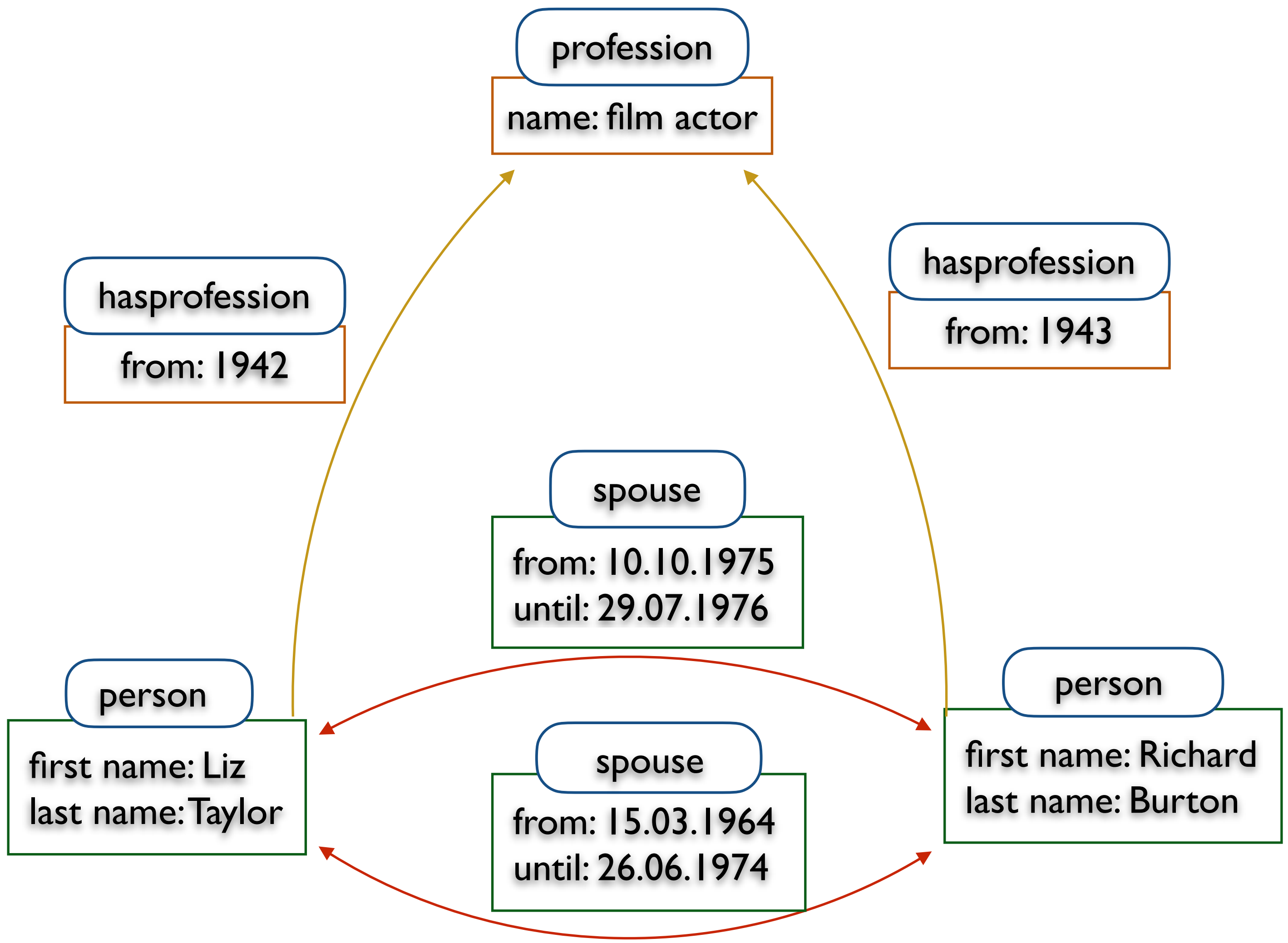

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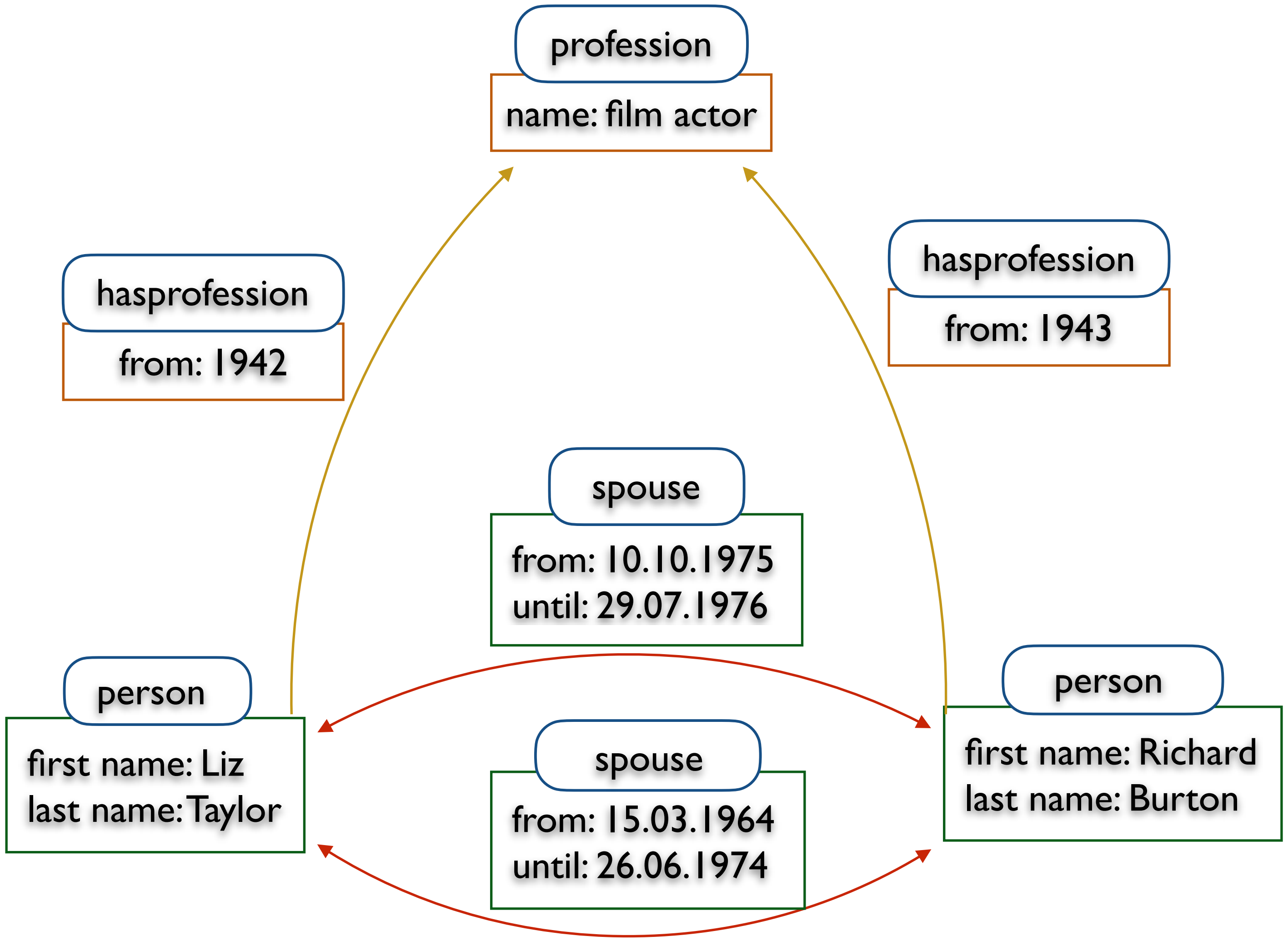
Property Graph Data Model



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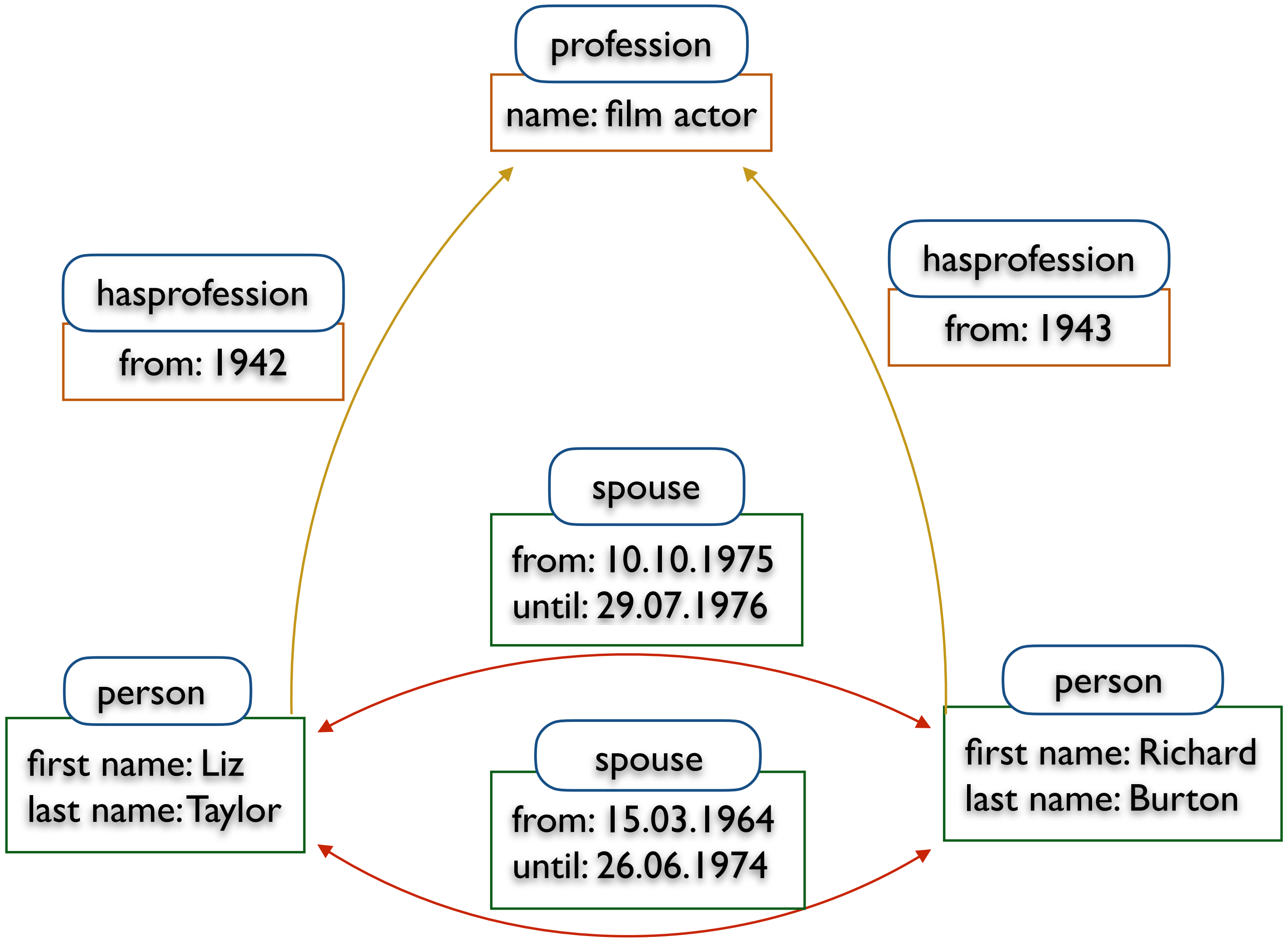

More formally, this is

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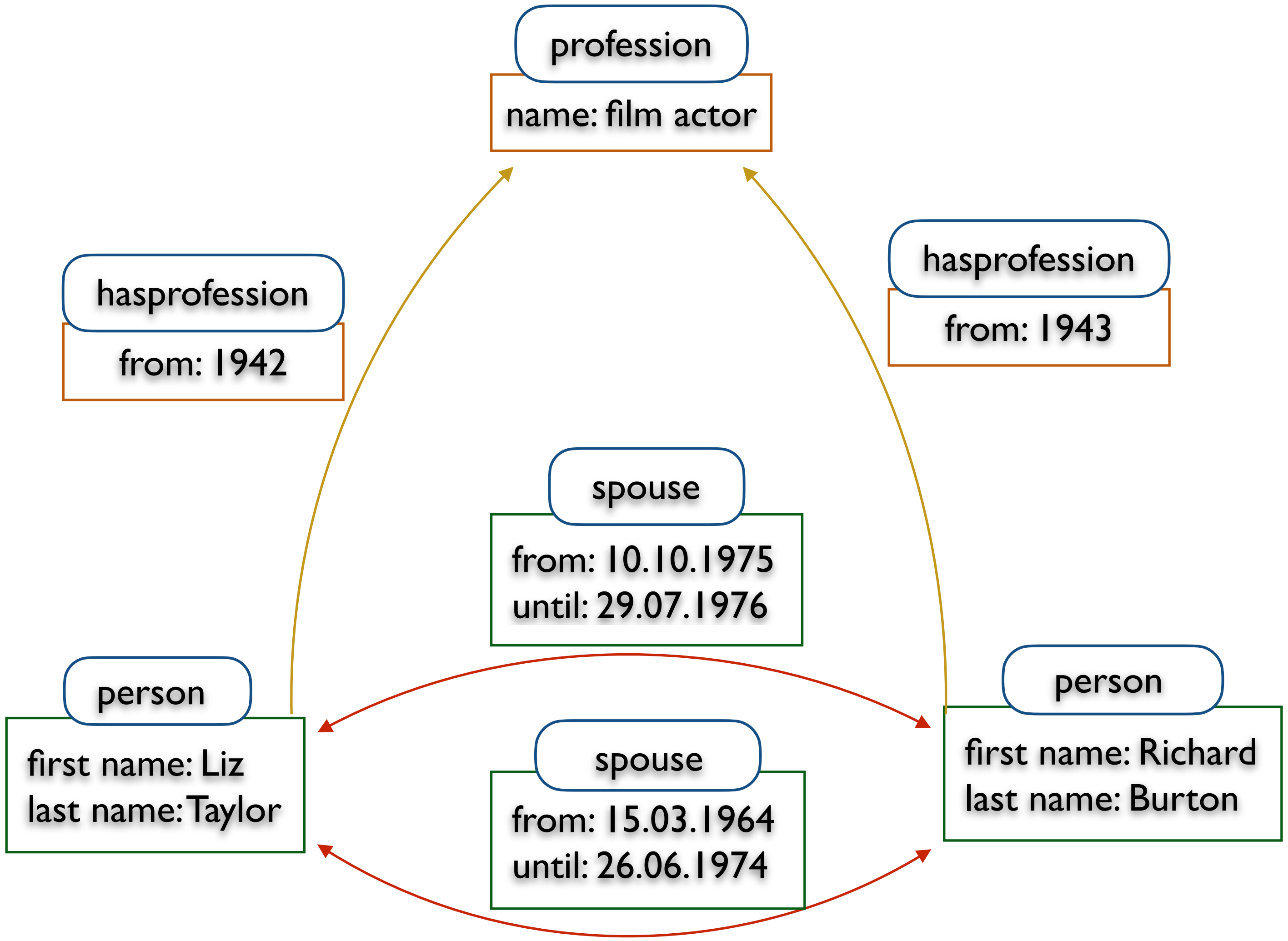
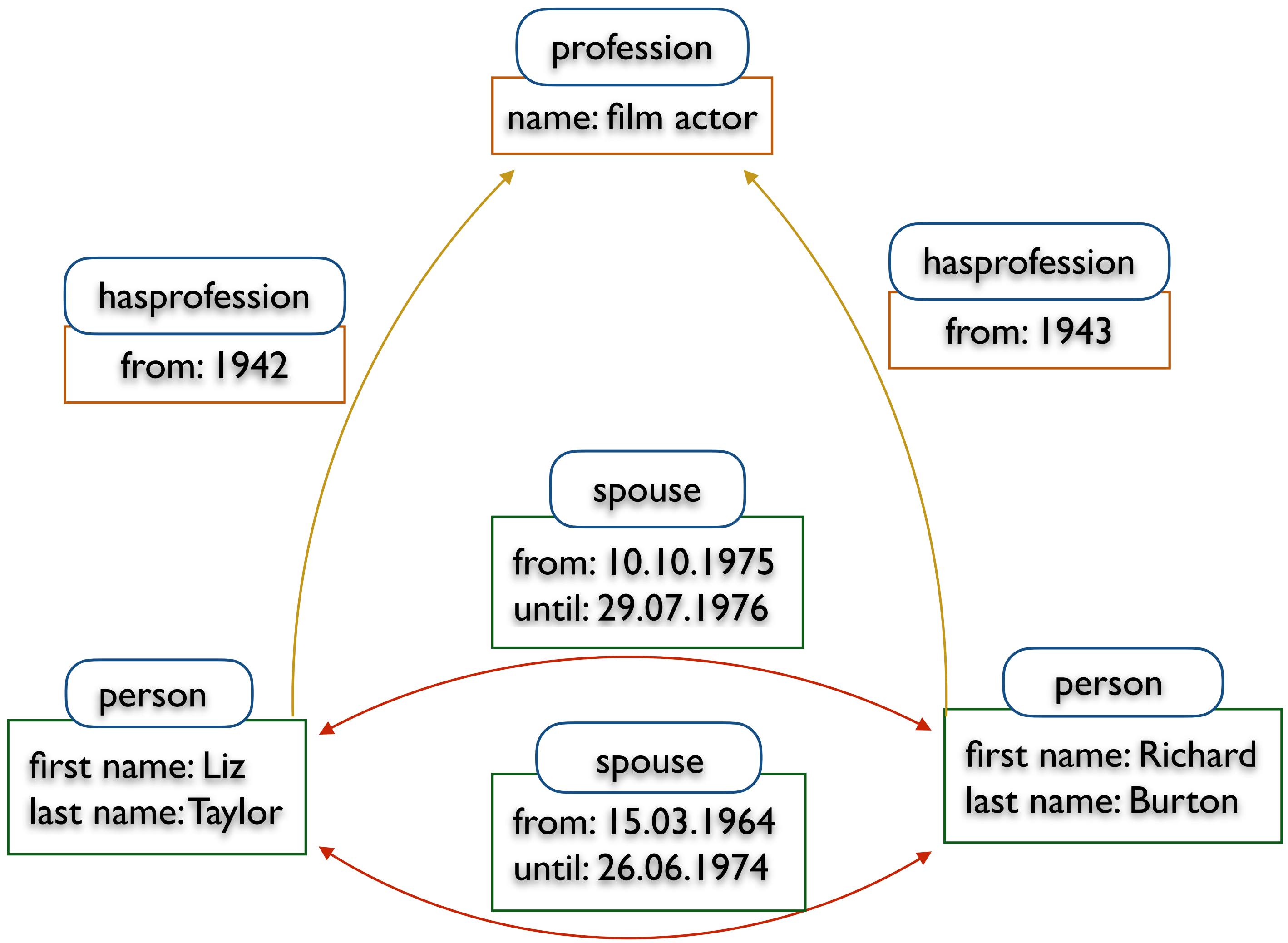


More formally, this is

- a set of node identifiers N
- a set of edge identifiers E
- a function that maps E to N × N

Property Graph Data Model



Labels L: person, profession, spouse

More formally, this is

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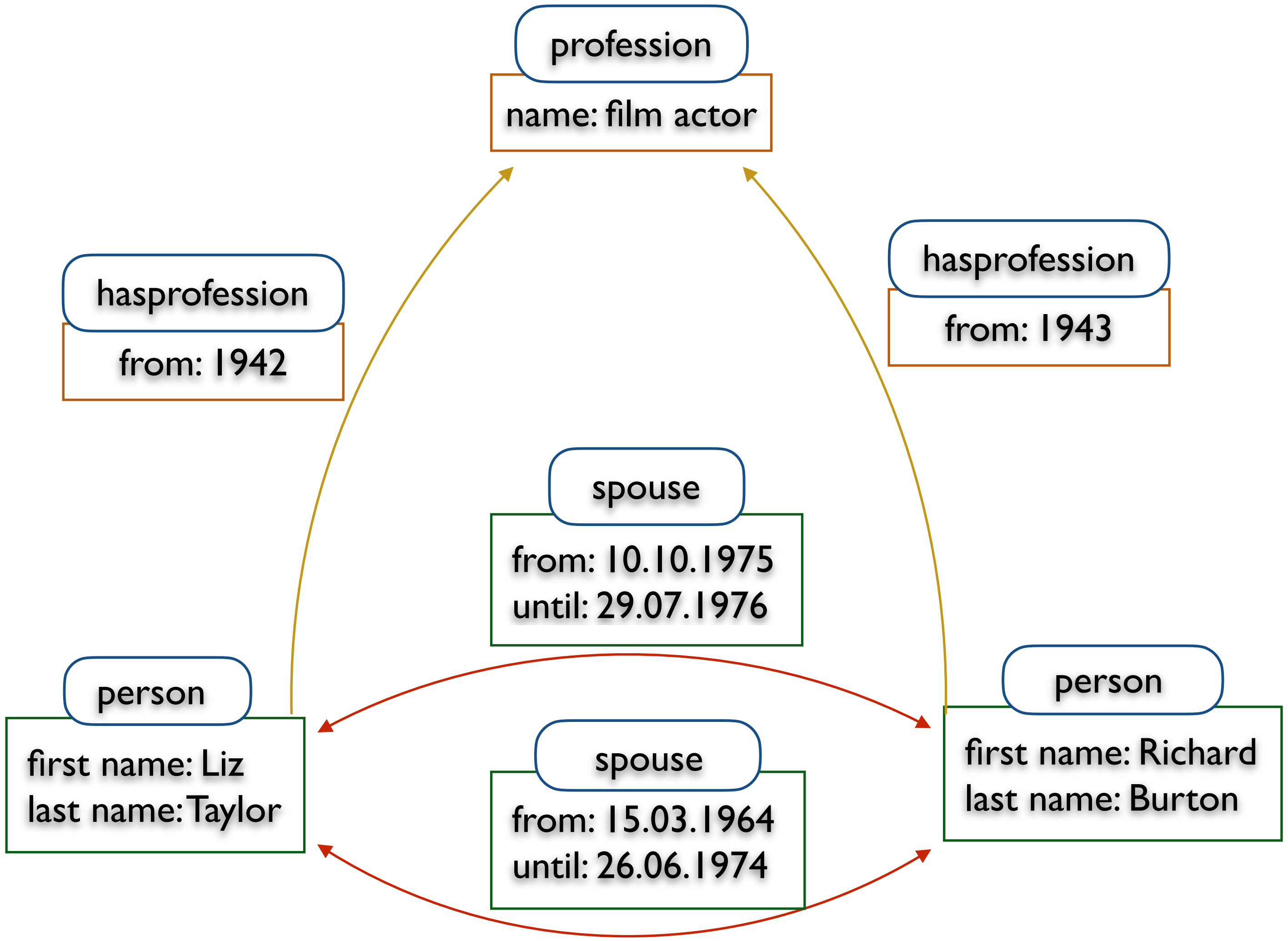

Labels **L**: person, profession, spouse

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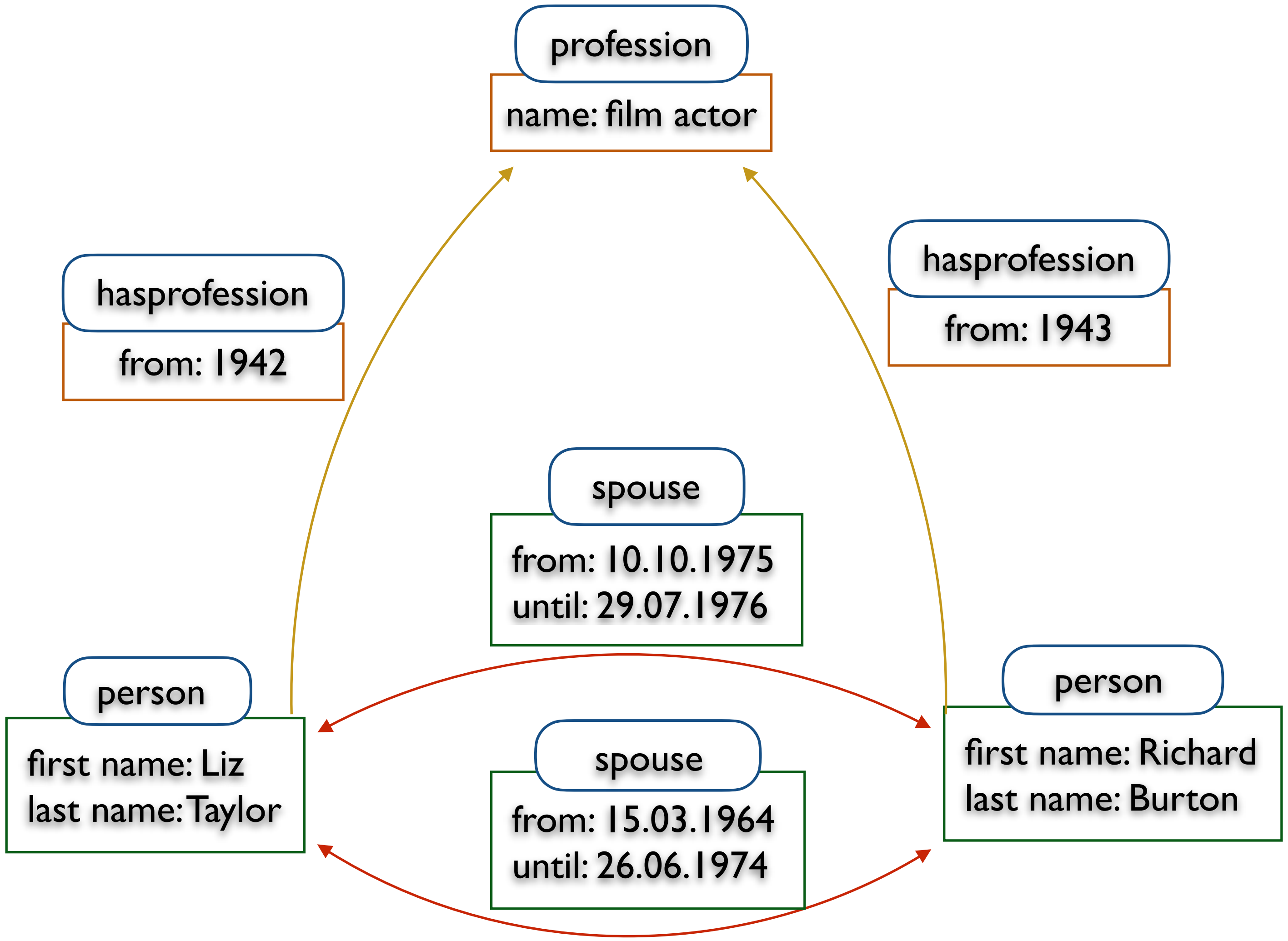

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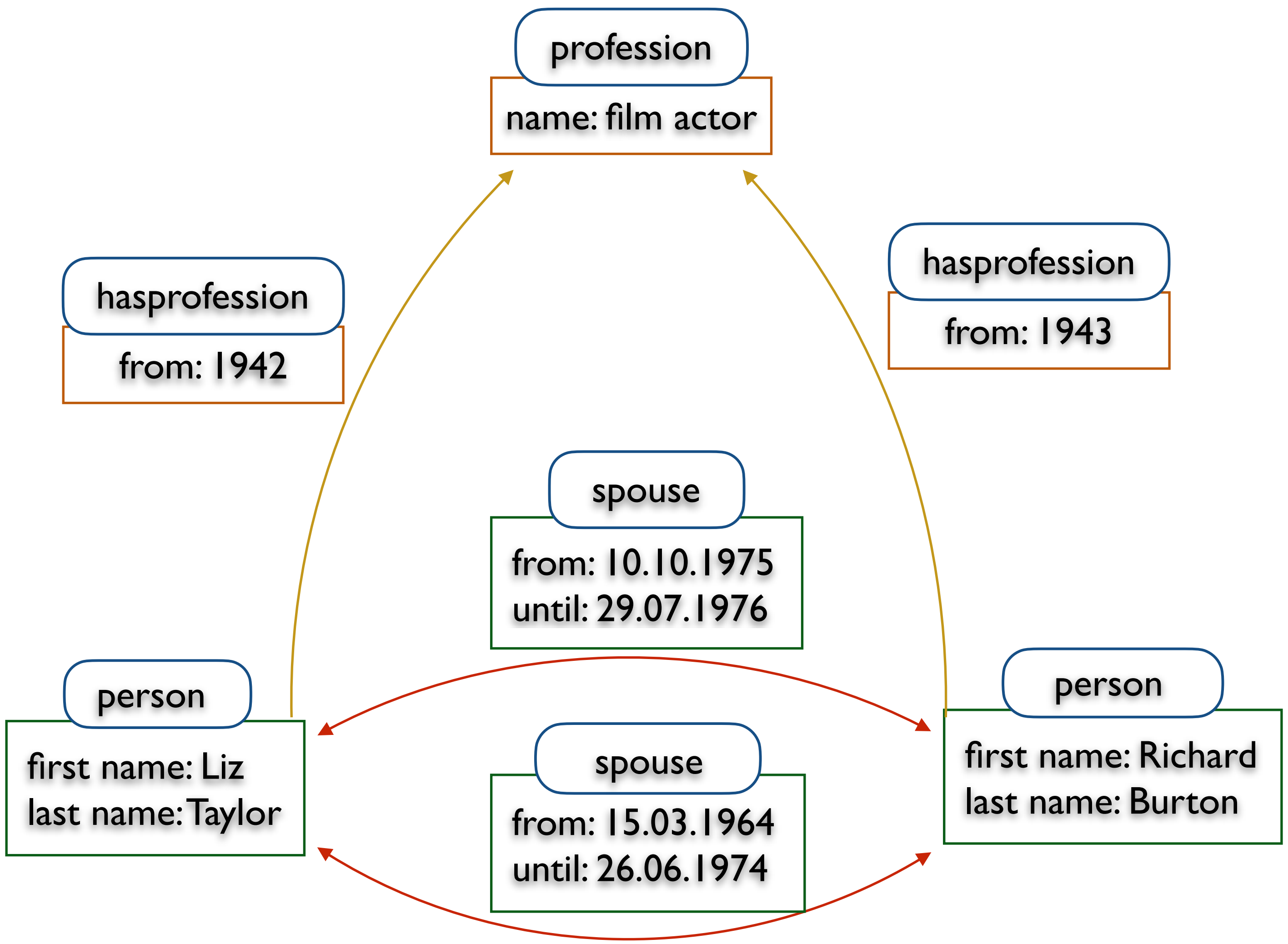

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- a function that maps E to N × N
- a function from N ∪ E to (subsets of) labels **L**

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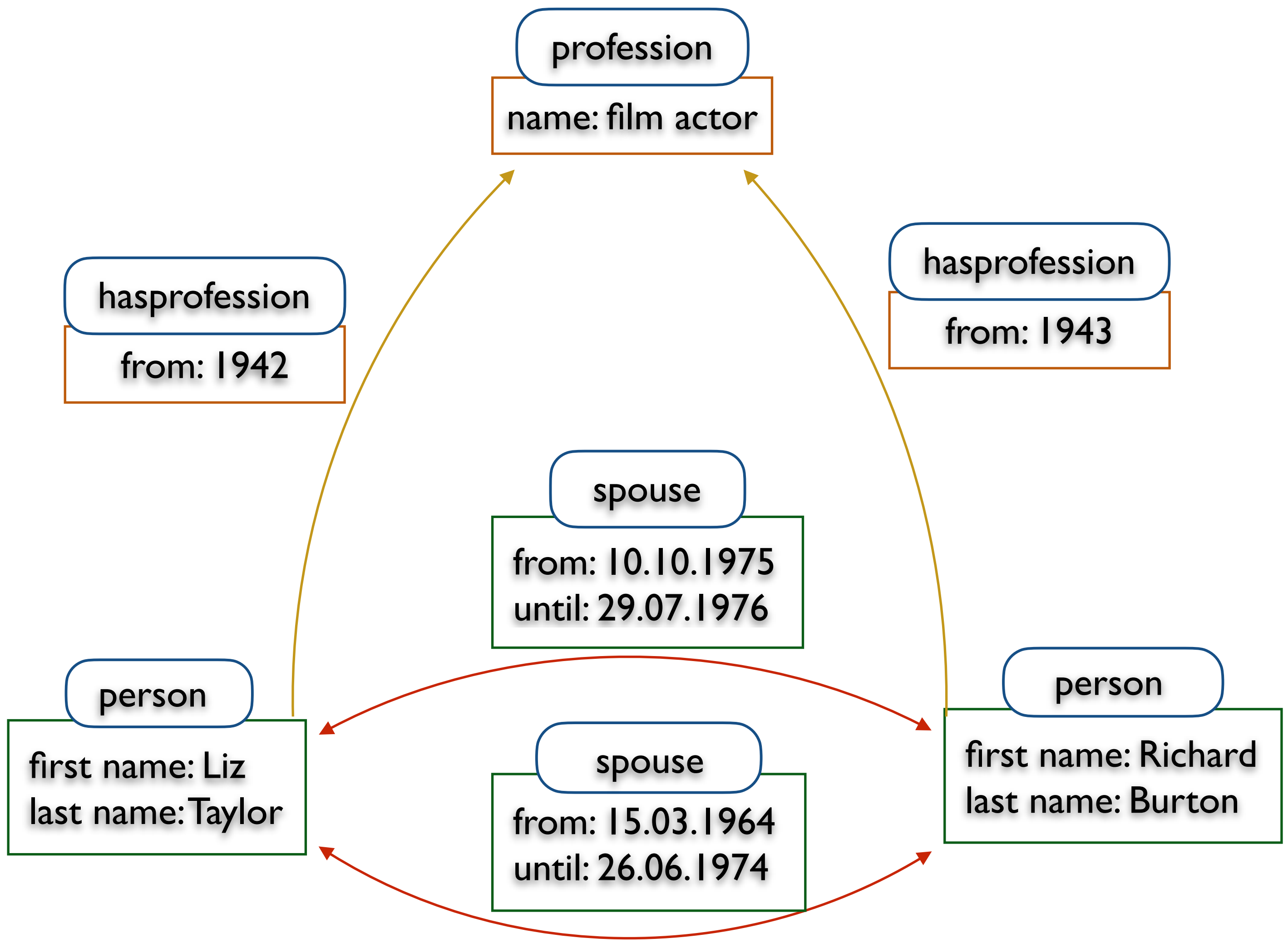

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More formally, this is

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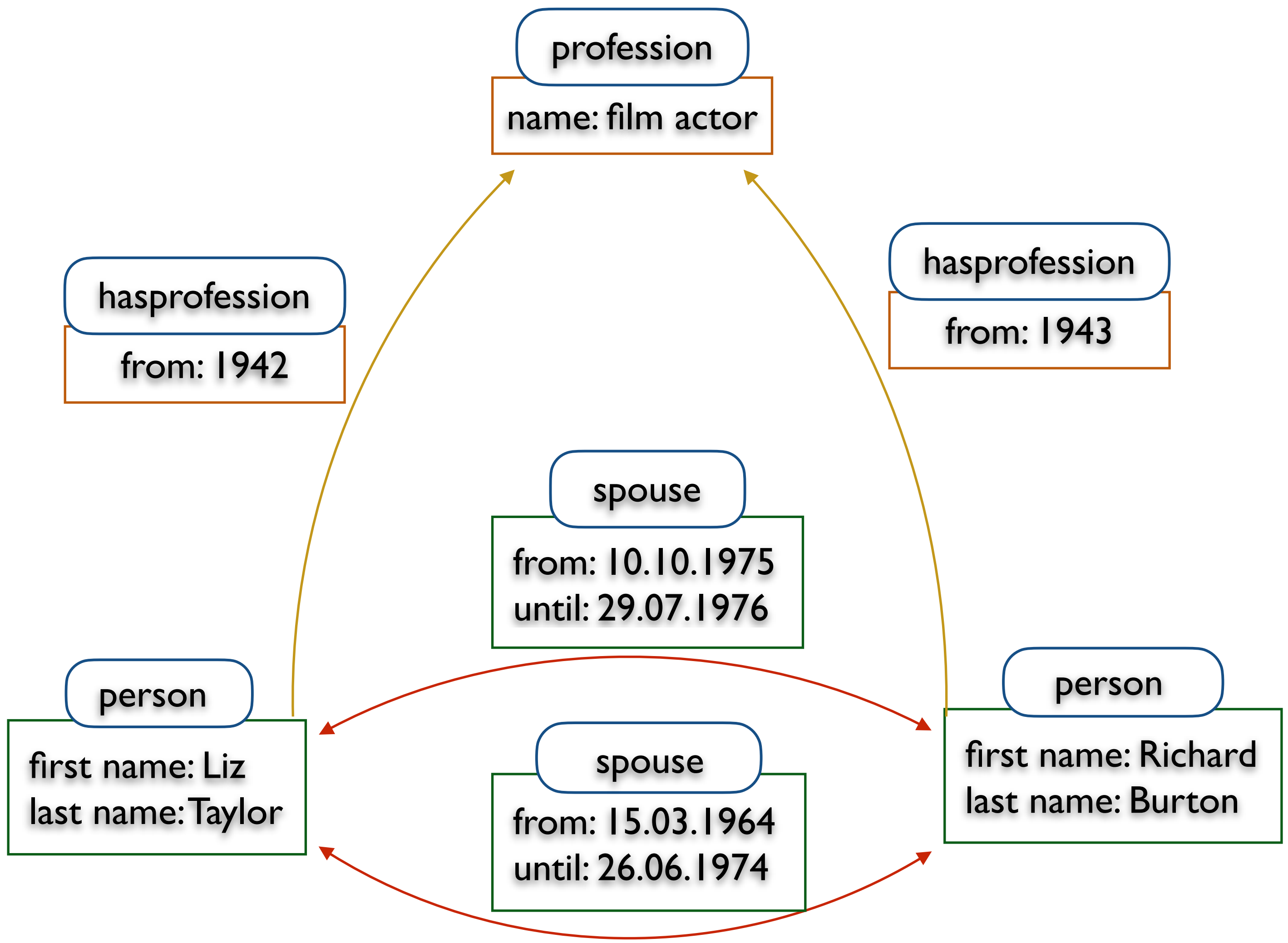
- a function that maps E to $N \times N$
- a function from $N \cup E$ to (subsets of) labels **L**
- a function from $(N \cup E) \times \mathbf{P}$ to (subsets of) values **V**

Property Graph Data Model



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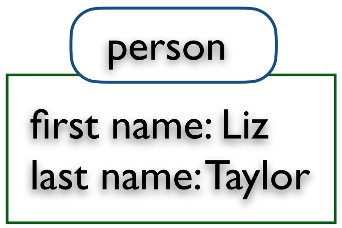
Some models also directly incorporate paths

RDF Data Model



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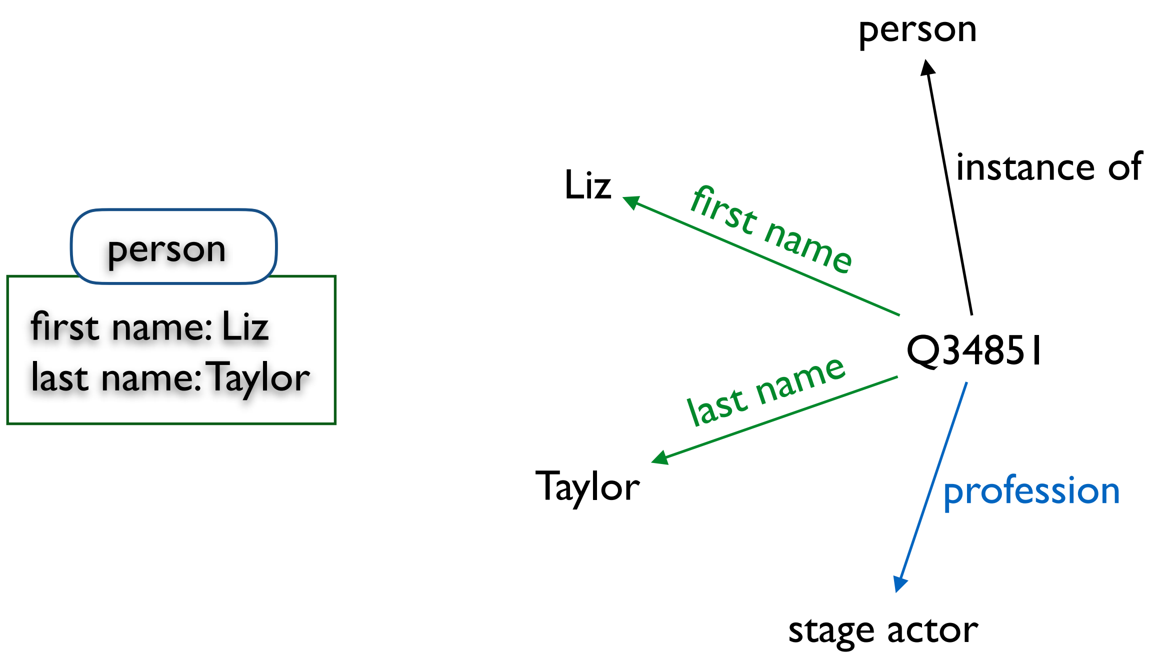

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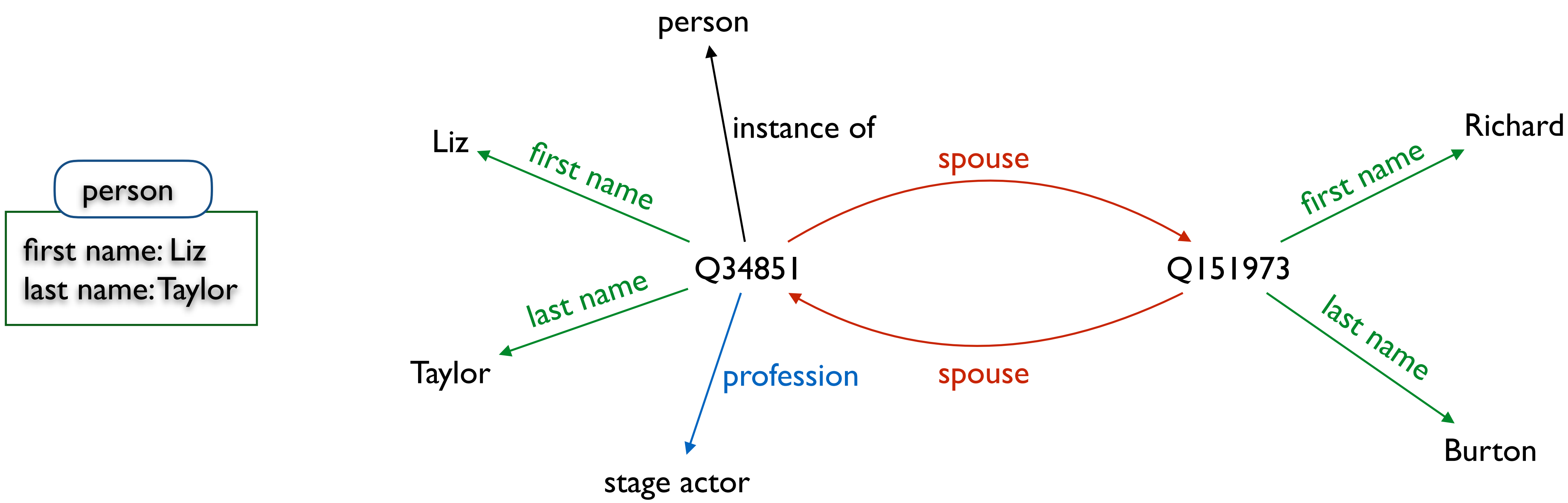

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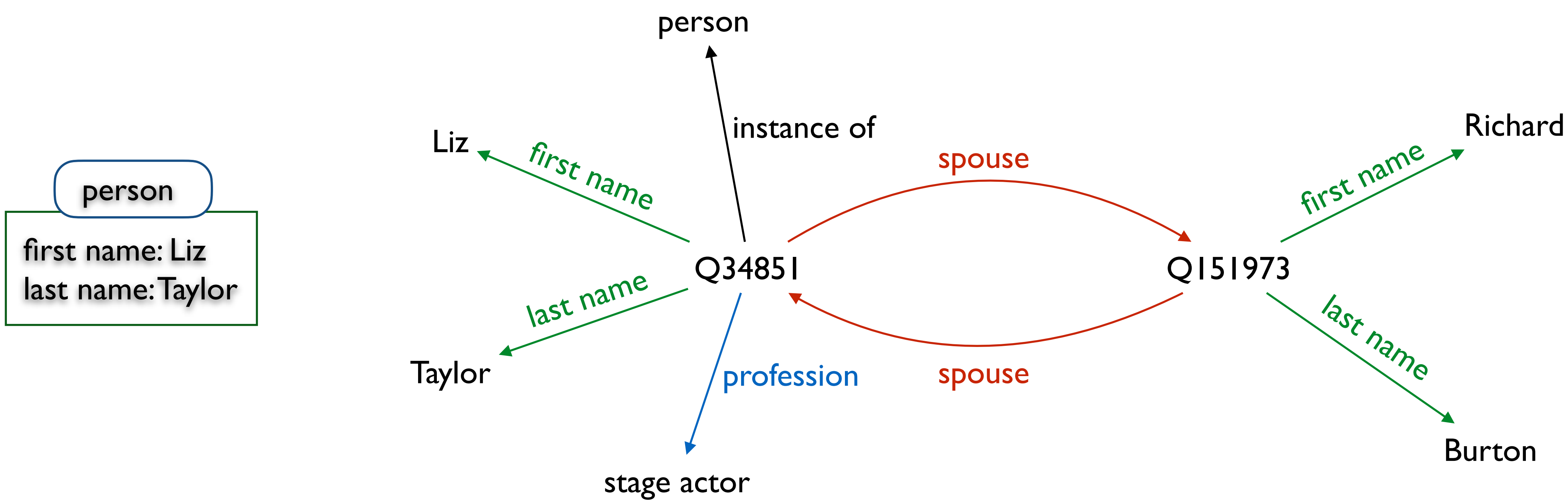

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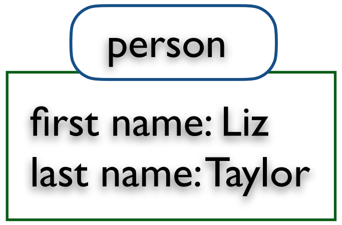

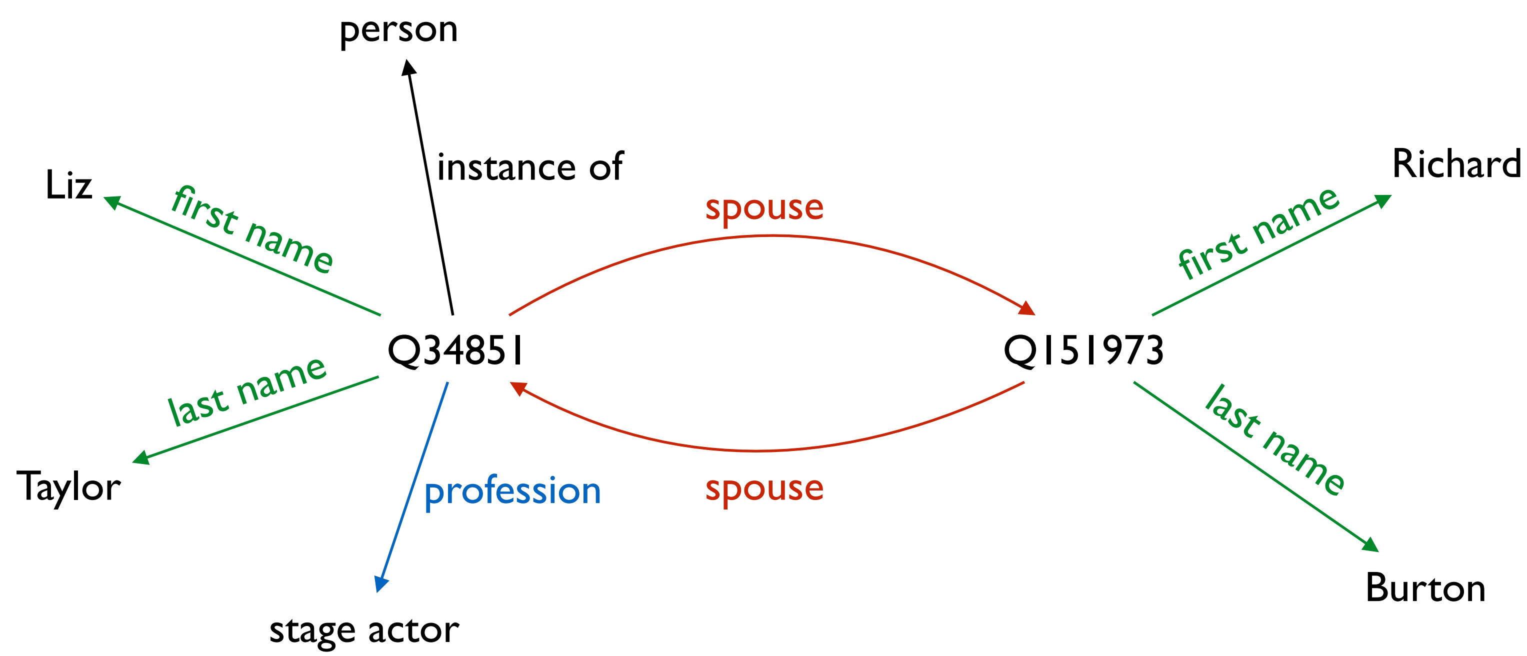

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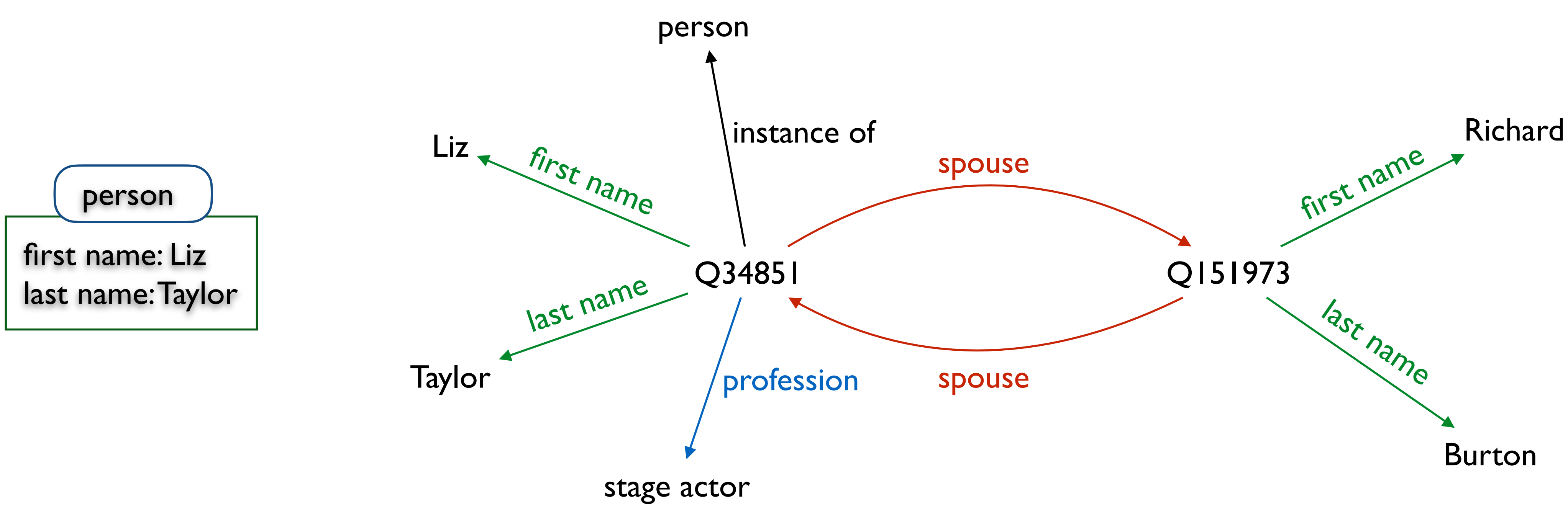
$$I \times I \times (I \cup L)$$

RDF Data Model



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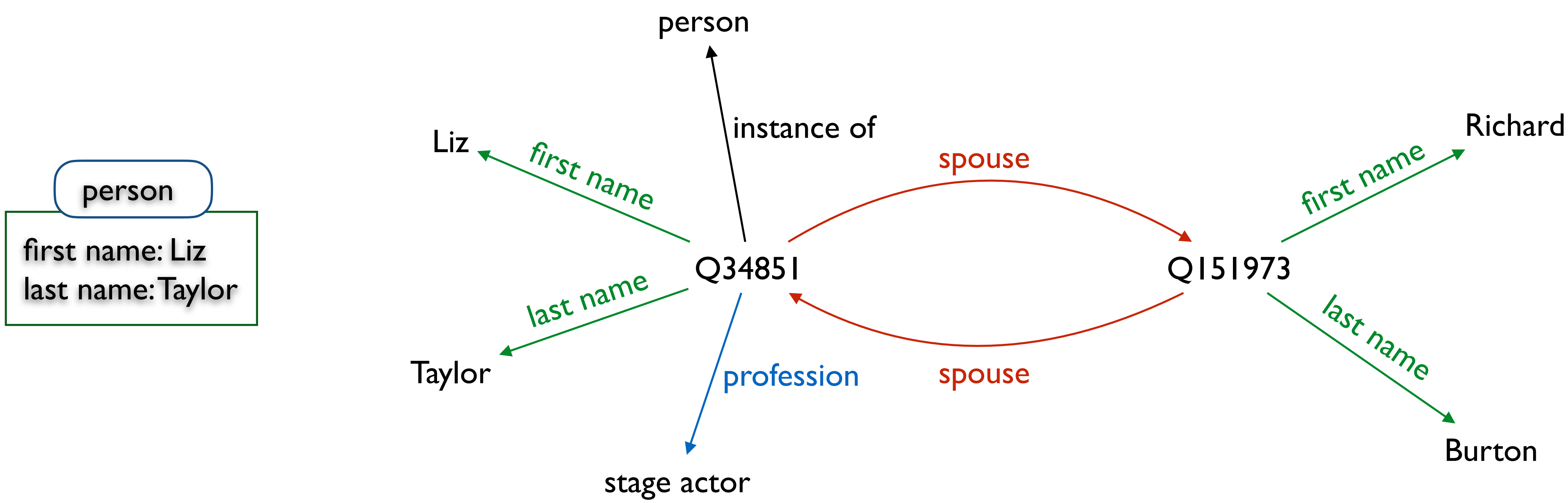

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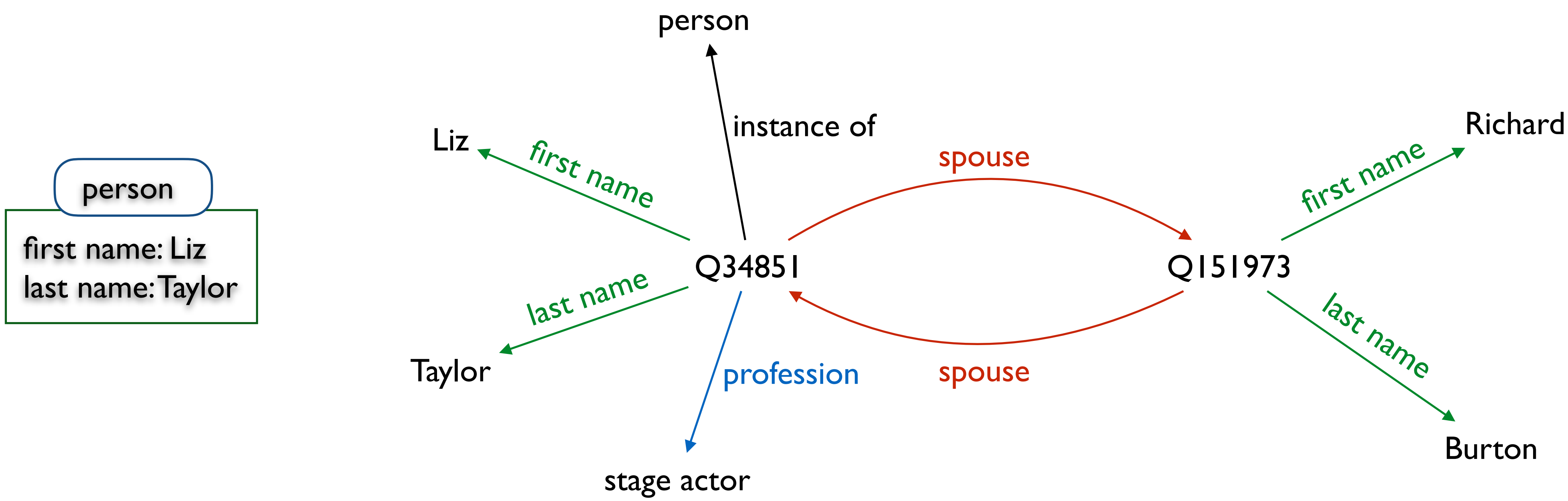
RDF Data Model



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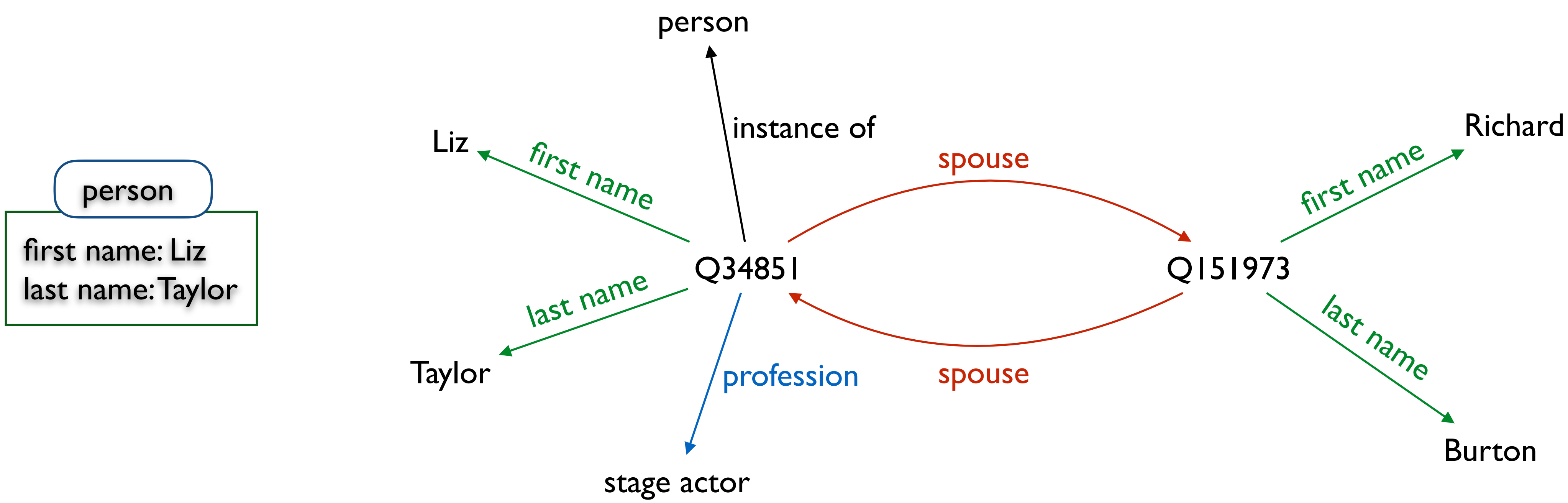

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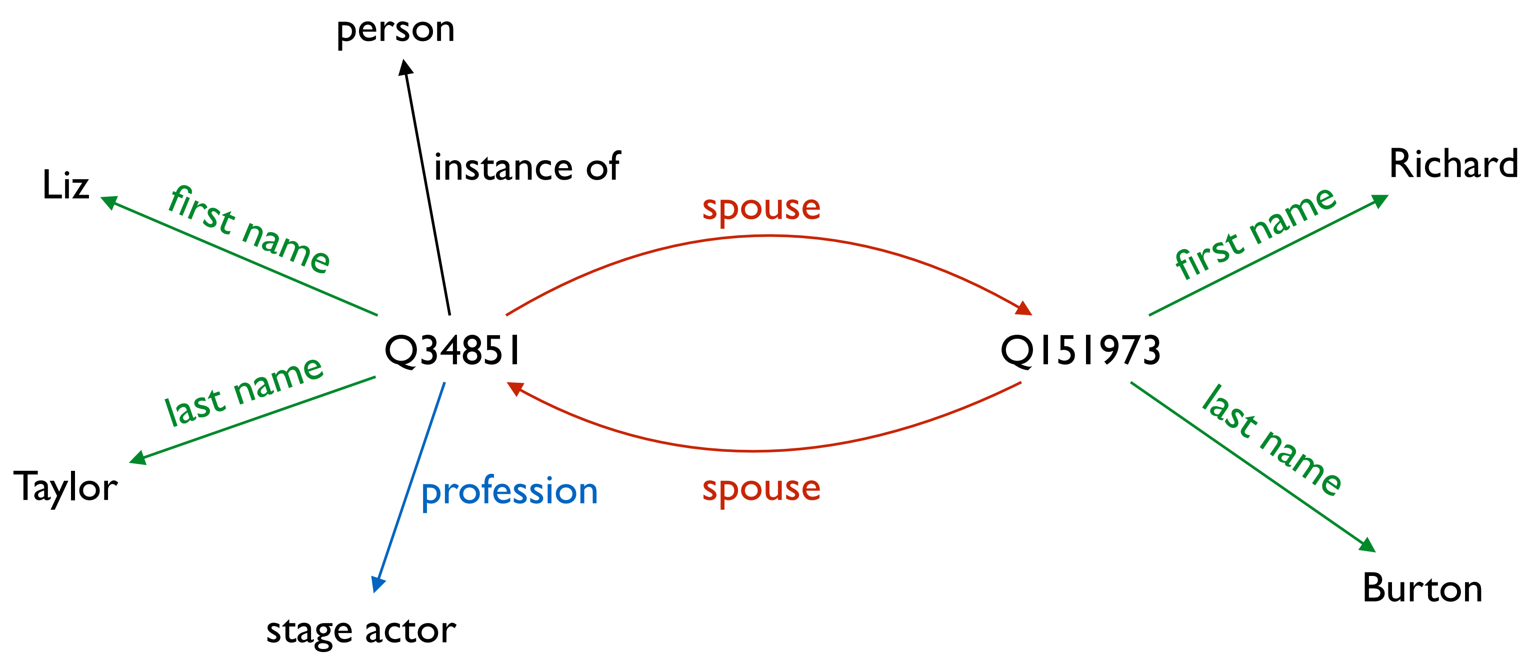
These triples (s,p,o) are referred to as subject / predicate / object triples

Most theoretical development is based on



Edge-labeled, directed graphs

Graph Database

We assume that Σ is a countably infinite set of labels

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Definition

A **graph database** (over Σ) is a pair $G = (V, E)$ where

- V is a finite set of nodes
- $E \subseteq V \times \Sigma \times V$ is a finite set of edges

Building blocks of query languages: RPQs and CRPQs

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Conjunctive Queries (CQs)

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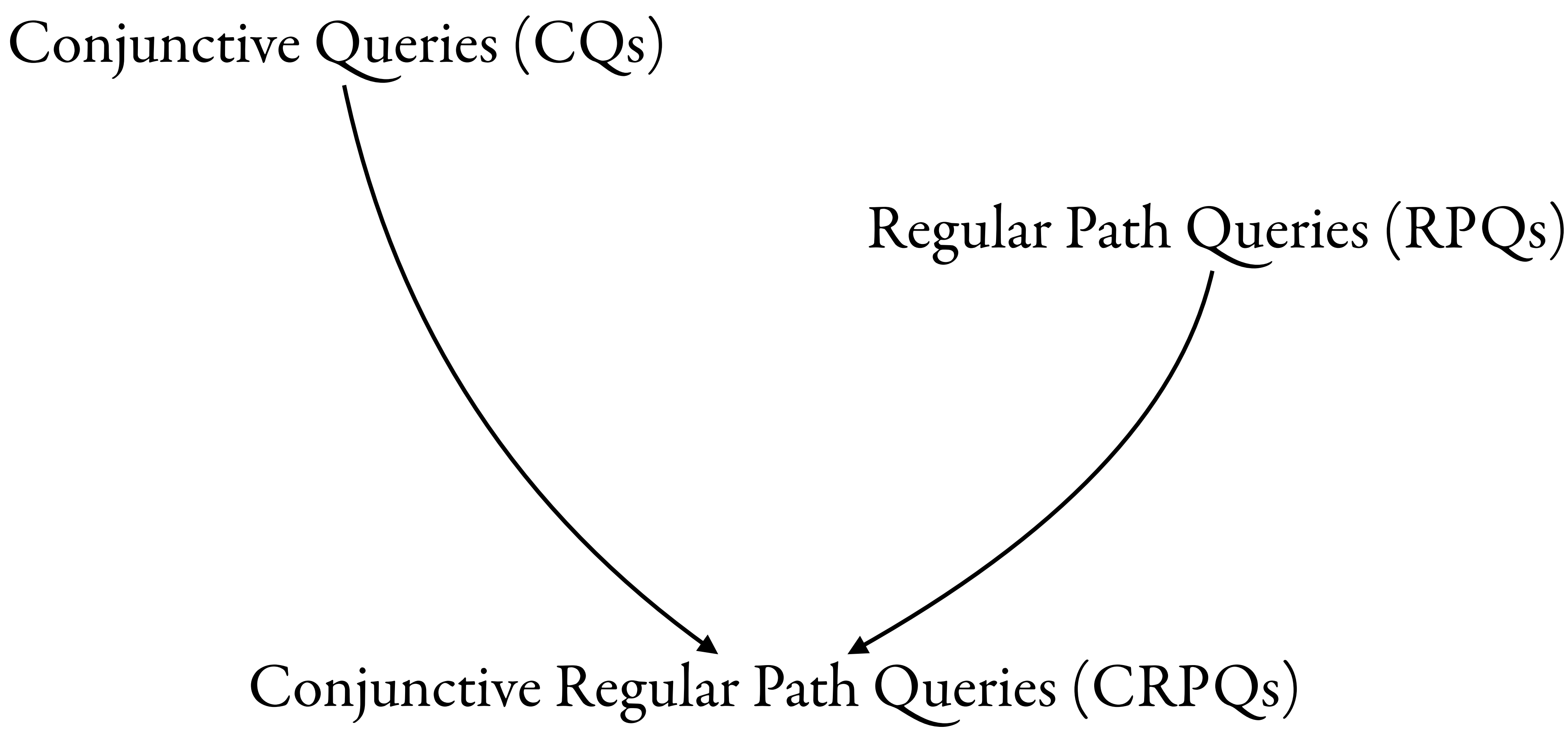
Conjunctive Queries (CQs)

Regular Path Queries (RPQs)

Building blocks of query languages: RPQs and CRPQs



Notation and Basic Principles

If $n \in \mathbb{N}$, we use $[n]$ to denote the set $\{1, \dots, n\}$

Regular Expressions

Operators:

(1)	Kleene star	(denoted *)
(2)	concatenation	(omitted in notation)
(3)	disjunction	(denoted +)

Priorities of operators: first (1), then (2), then (3)

Example: $ab + cd^*$

The **language of regular expression r** is denoted $L(r)$

We use r^n to abbreviate n-fold concatenation of r

(So we write a^4 for $aaaa$)

Regular Path Queries

Why regular path queries?

Conjunctive queries (and even first-order queries) on graphs are limited:

they can only express local properties

Regular path queries overcome this, using regular expressions to query paths

Definition

A path in graph G is a sequence

$$p = (v_0, a_1, v_1)\,(v_1, a_2, v_2)\,\ldots\,(v_{n\text{-}2}, a_n, v_{n\text{-}1})\,(v_{n\text{-}1}, a_n, v_n)$$

of edges of G. Label of p is $a_1 a_2 \cdots a_n$

Regular Path Queries

Definition

A **regular path query (RPQ)** is an expression of the form

$$x \xrightarrow{r} y$$

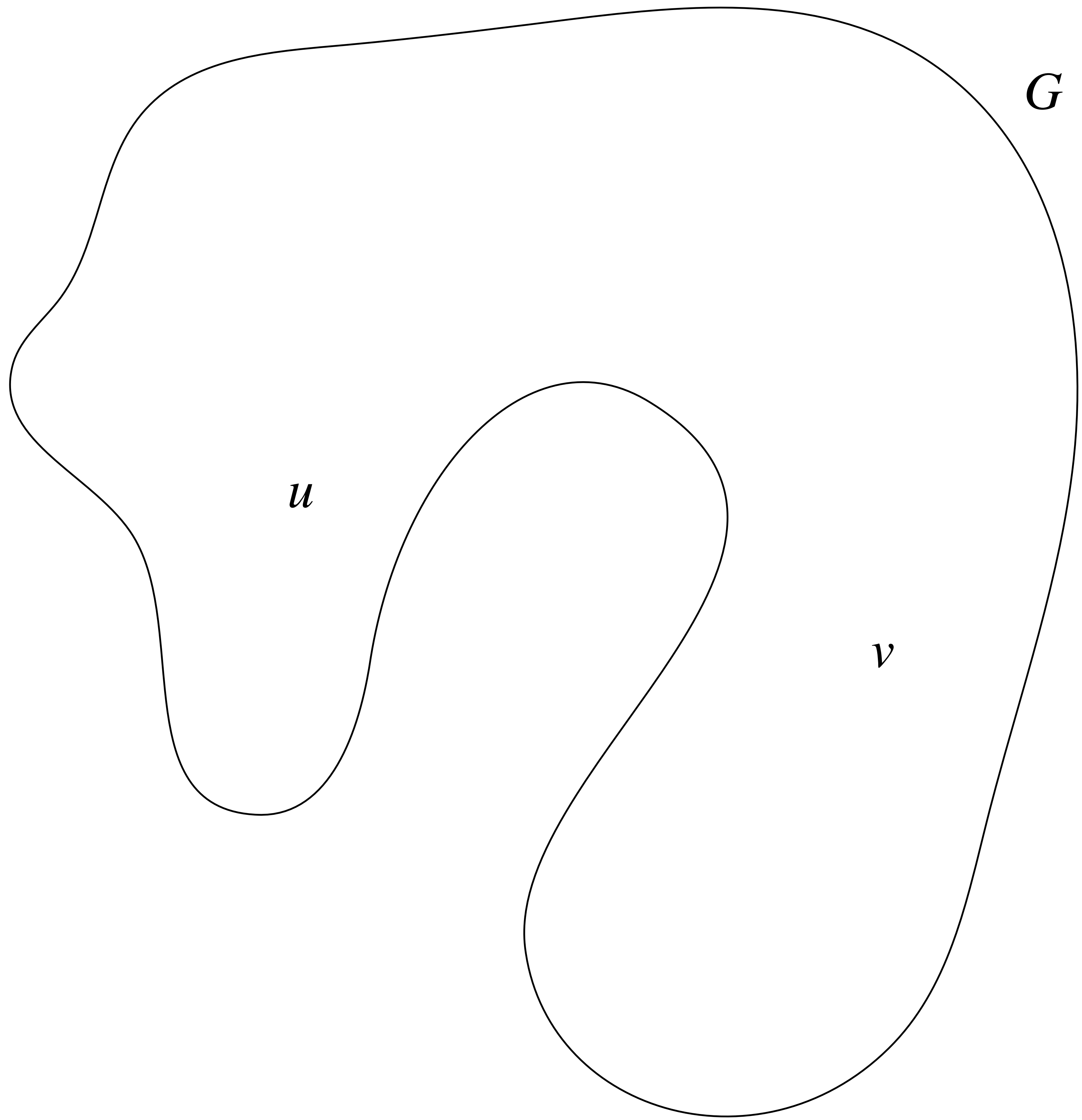
where x and y are variables and r is a regular expression over Σ

(Notice that r can only mention a finite subset of Σ)

Semantics of RPQs

RPQ

$x \xrightarrow{r} y$



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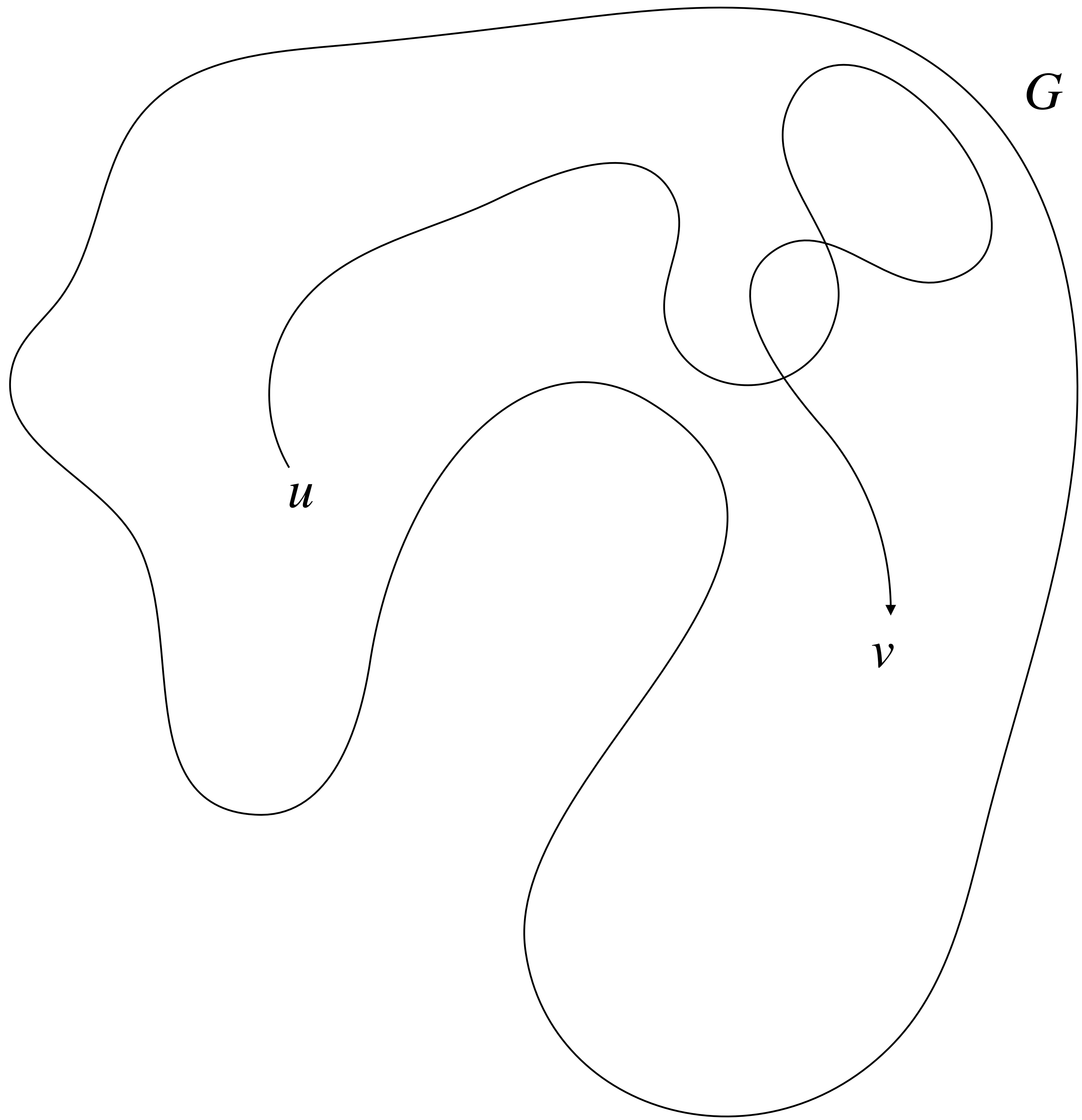

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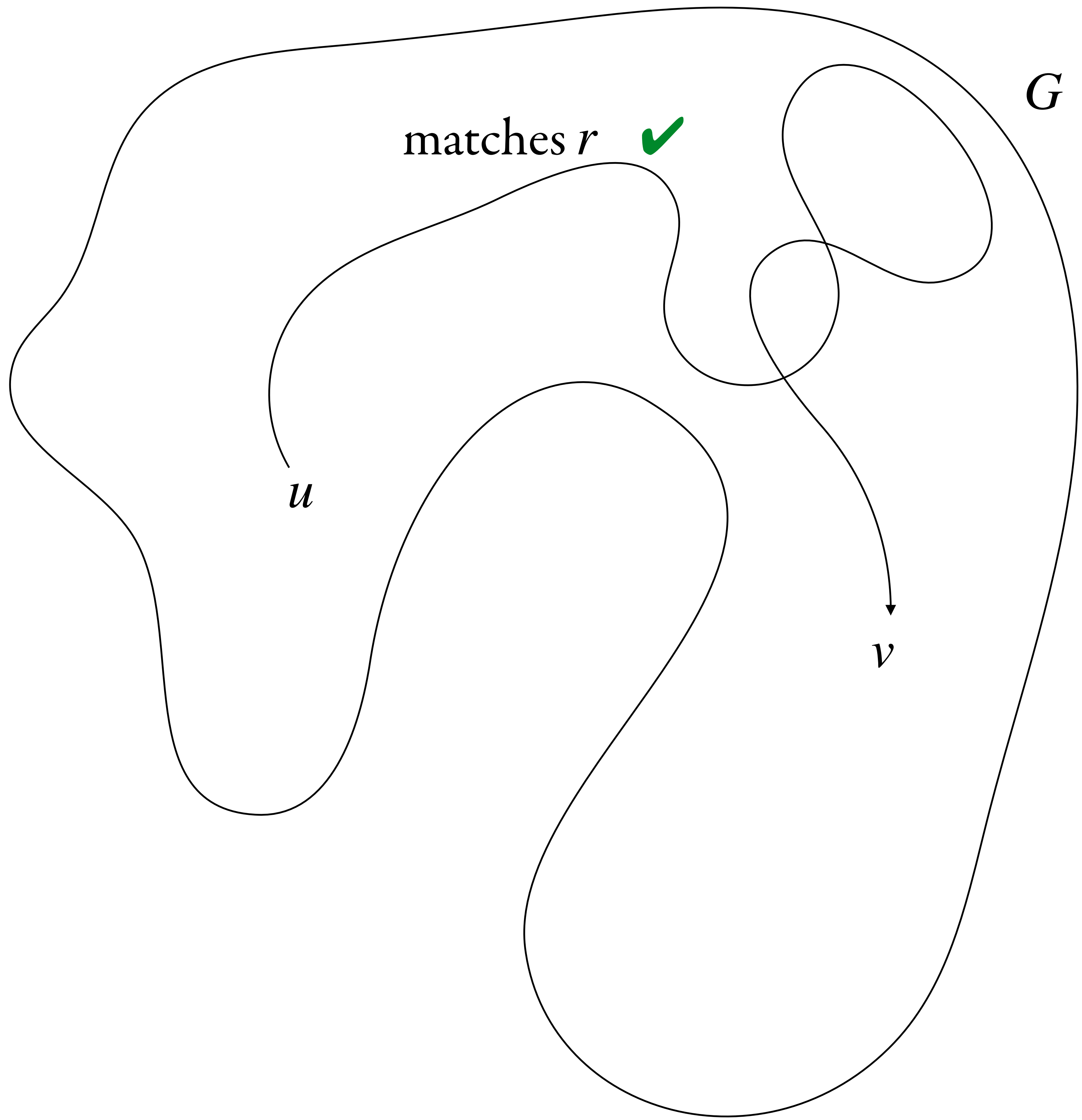

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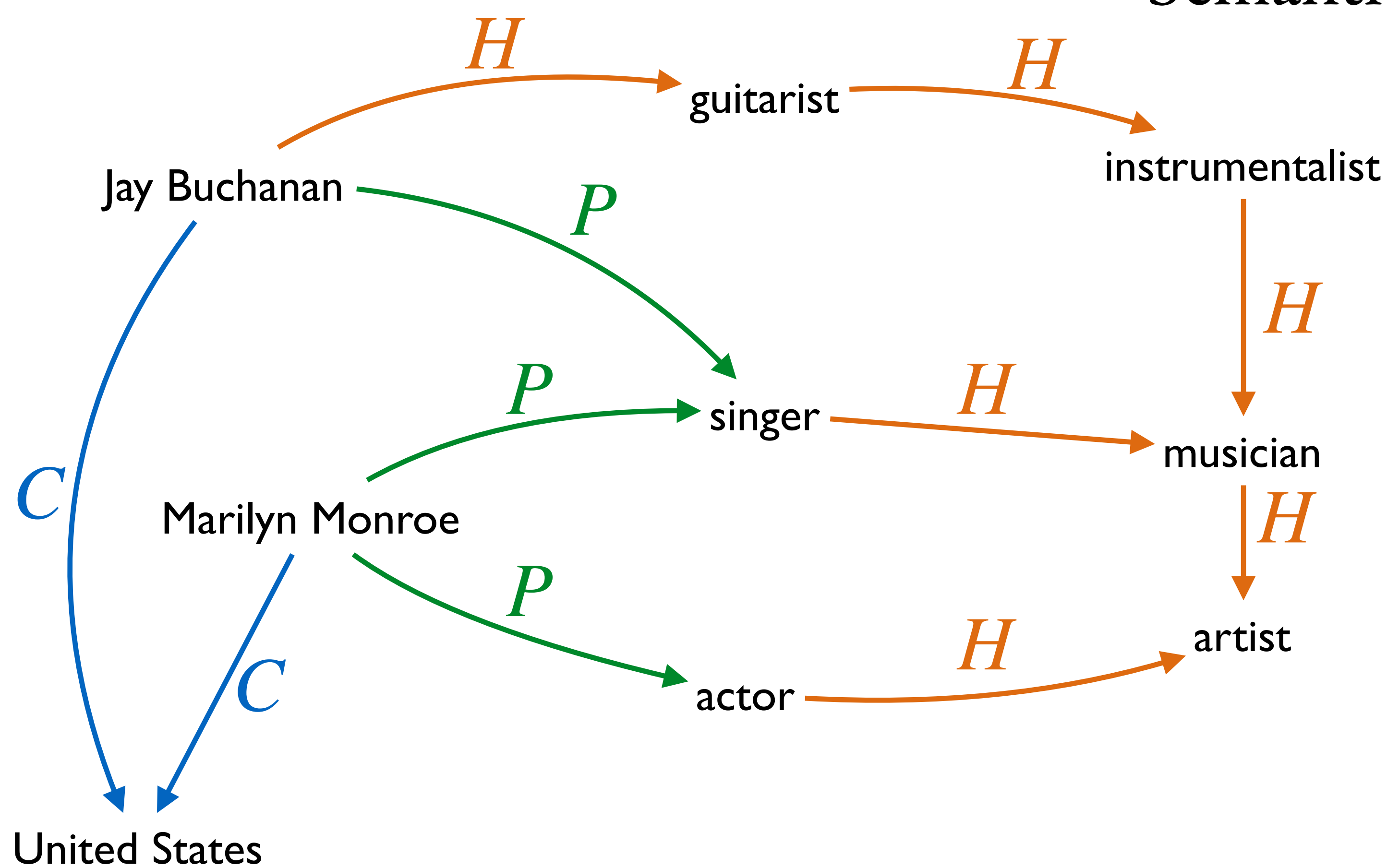
Regular Path Queries
Semantics



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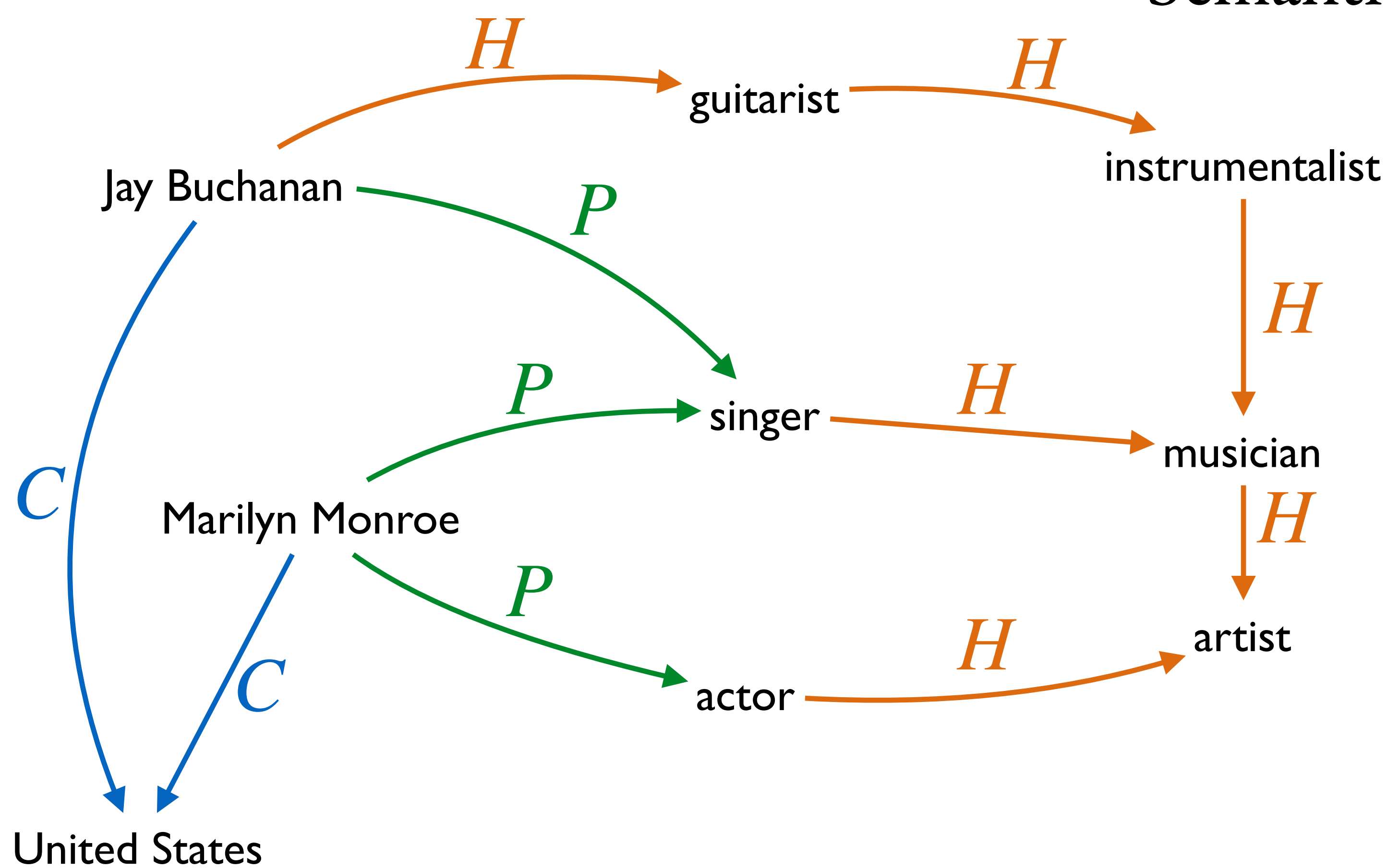

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Regular Path Queries

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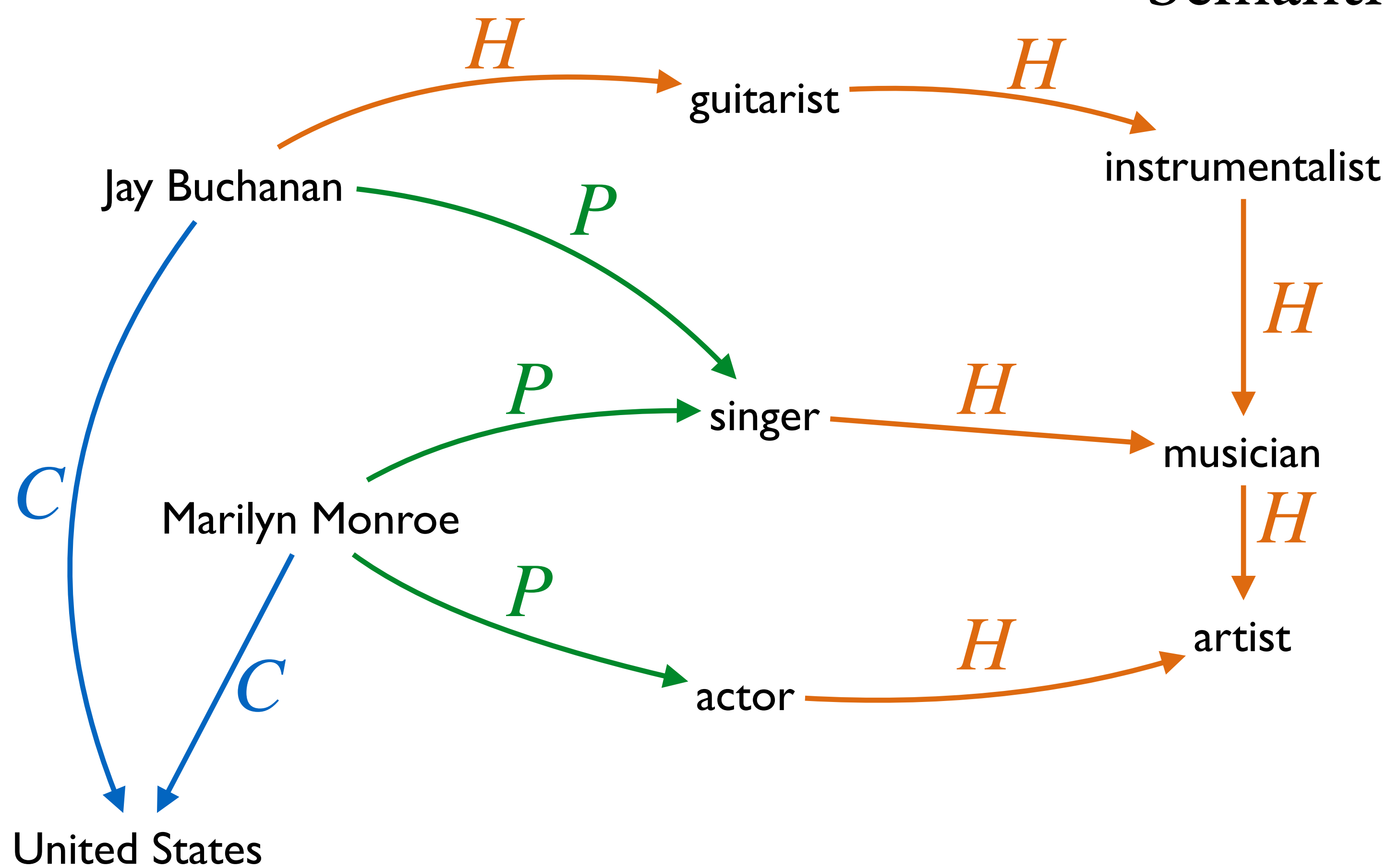

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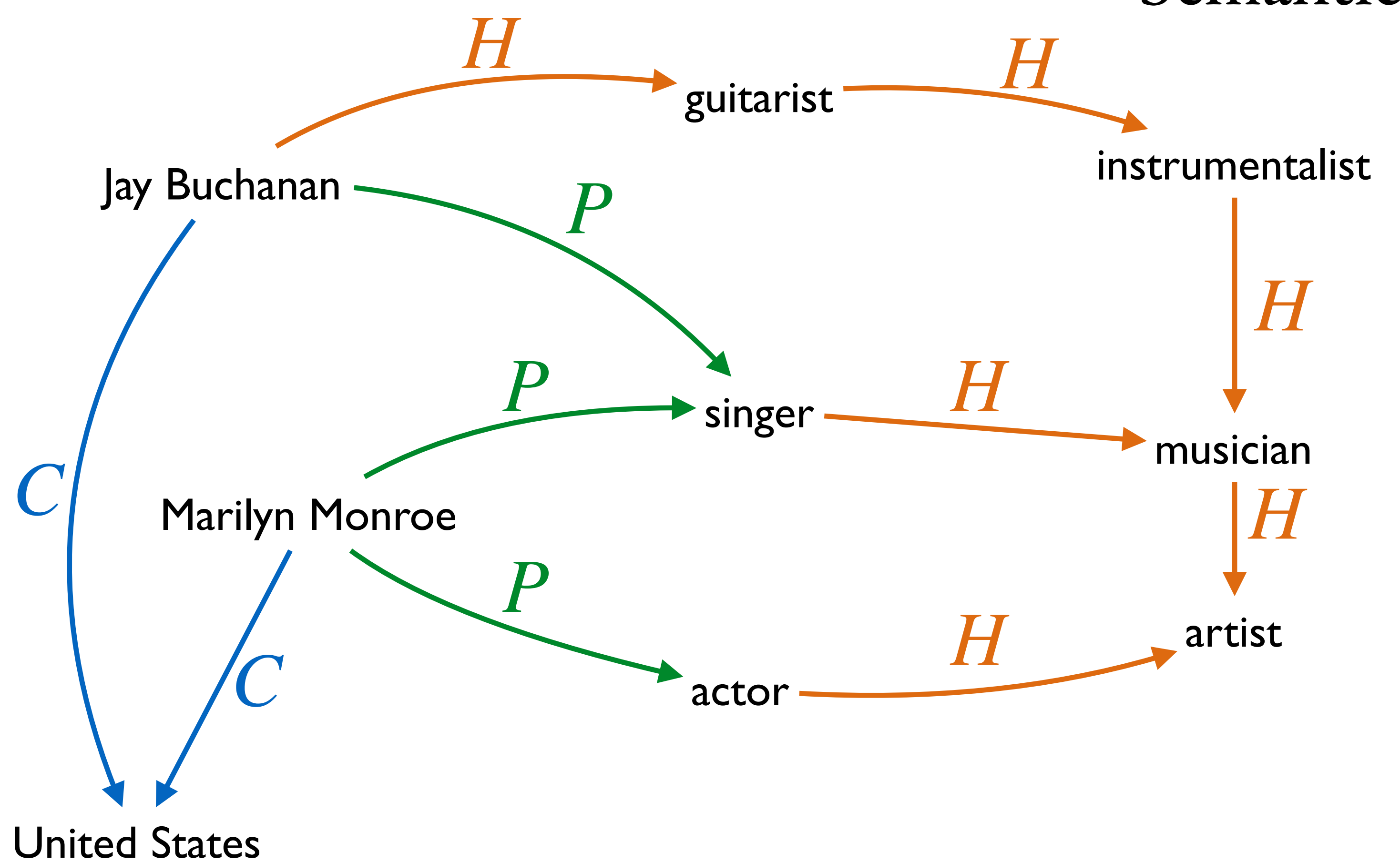

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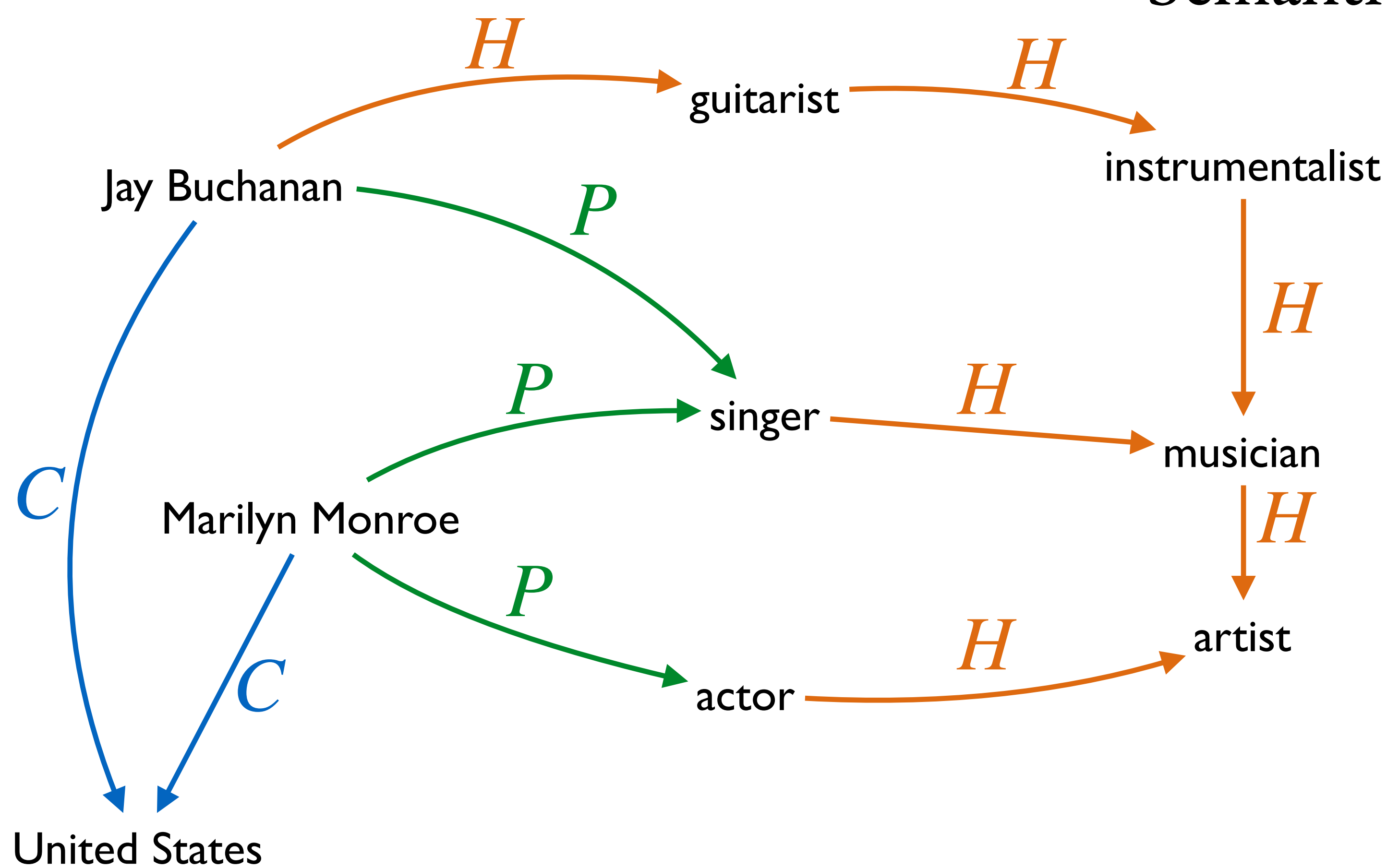

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Semantics of RPQs

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Matching Paths

Let r be a regular expression and G be a graph

A path $p = (v_0, a_1, v_1)\ (v_1, a_2, v_2)\ \dots\ (v_{n-1}, a_n, v_n)$ in G **matches** r, if

its label $a_1 a_2\ \dots\ a_n \in L(r)$

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Let $Q = (x \xrightarrow{r} y)$ be a regular path query and $G = (V, E)$ be a graph

The **answer of Q on G** is

$$Q(G) = \{(u, v) \in V \times V \mid \text{there exists a path } p \text{ from } u \text{ to } v \text{ in } G \text{ that matches } r\}$$

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Notation

If $Q = (x \xrightarrow{r} y)$, we sometimes denote $Q(G)$ by $r(G)$

Semantics of RPQs

Matching Paths

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Semantics of RPQs (every path semantics)

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Regular Path Queries

Semantics

There are different semantics of RPQs in the literature and in graph database systems!

every path

trail

simple path

shortest path

The differences between these are significant

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What to use in new languages:

Consensus - all. Every path (walk), shortest, simple, trail.

Semantics of RPQs

Semantics of RPQs (every path semantics)

Let $Q = (x \xrightarrow{r} y)$ be a regular path query and $G = (V, E)$ be a graph

The **answer of Q on G under every path semantics** is

$$Q(G) = \{(u, v) \in V \times V \mid \text{there exists a path } p \text{ from } u \text{ to } v \text{ in } G \text{ that matches } r\}$$

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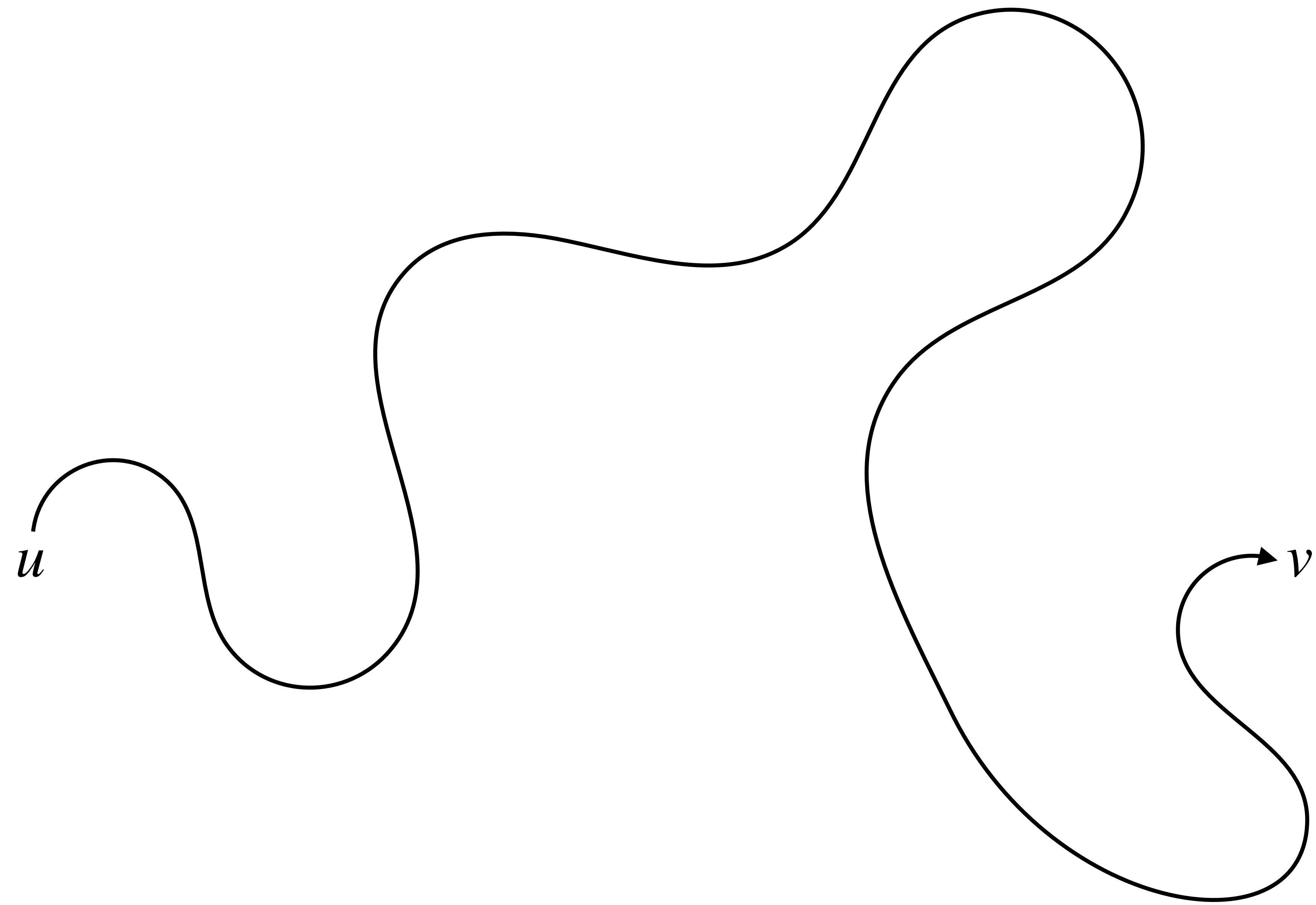
Hence, "every path" is eligible for the query

Simple Paths and Trails

u v

Simple Paths and Trails



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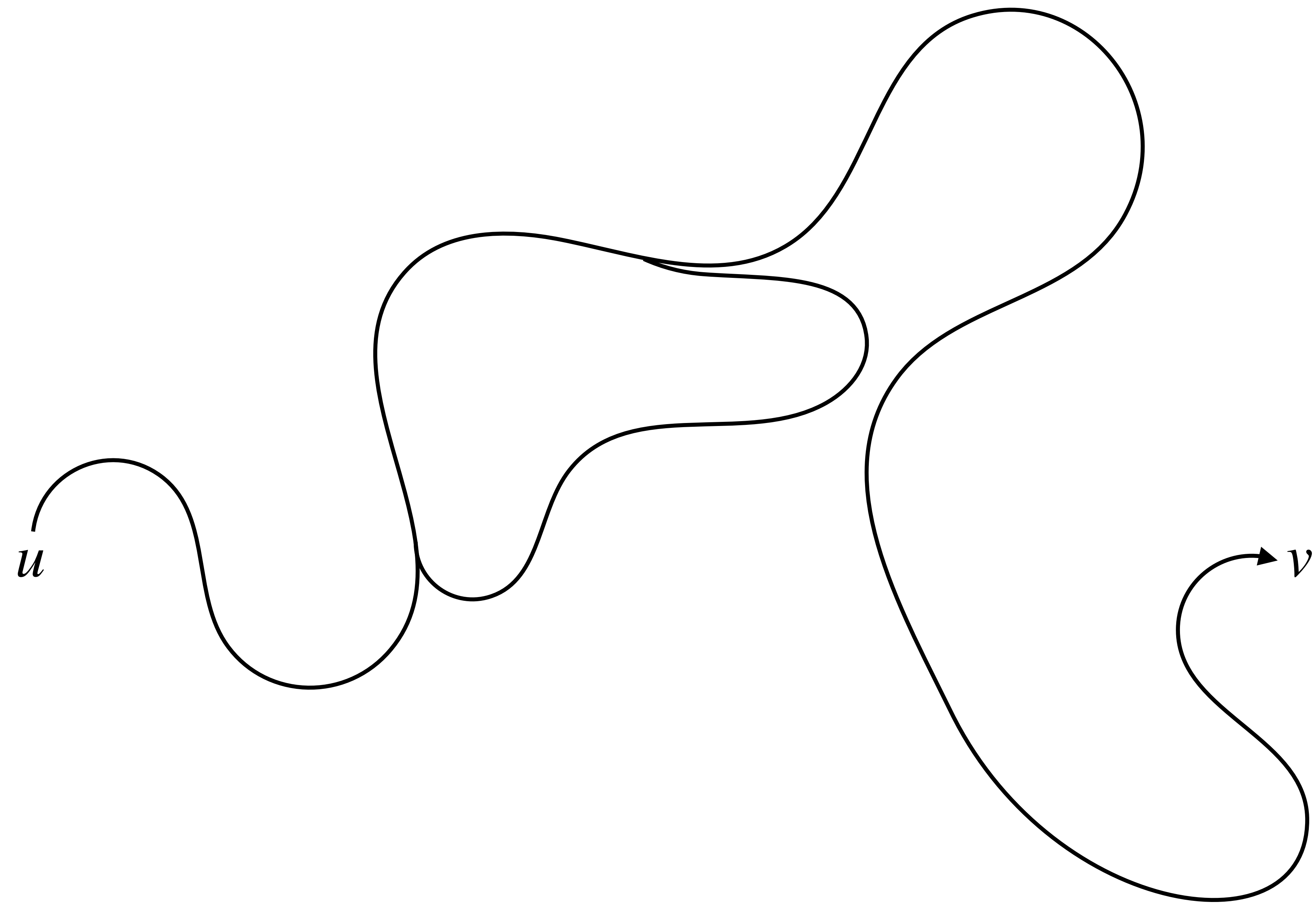

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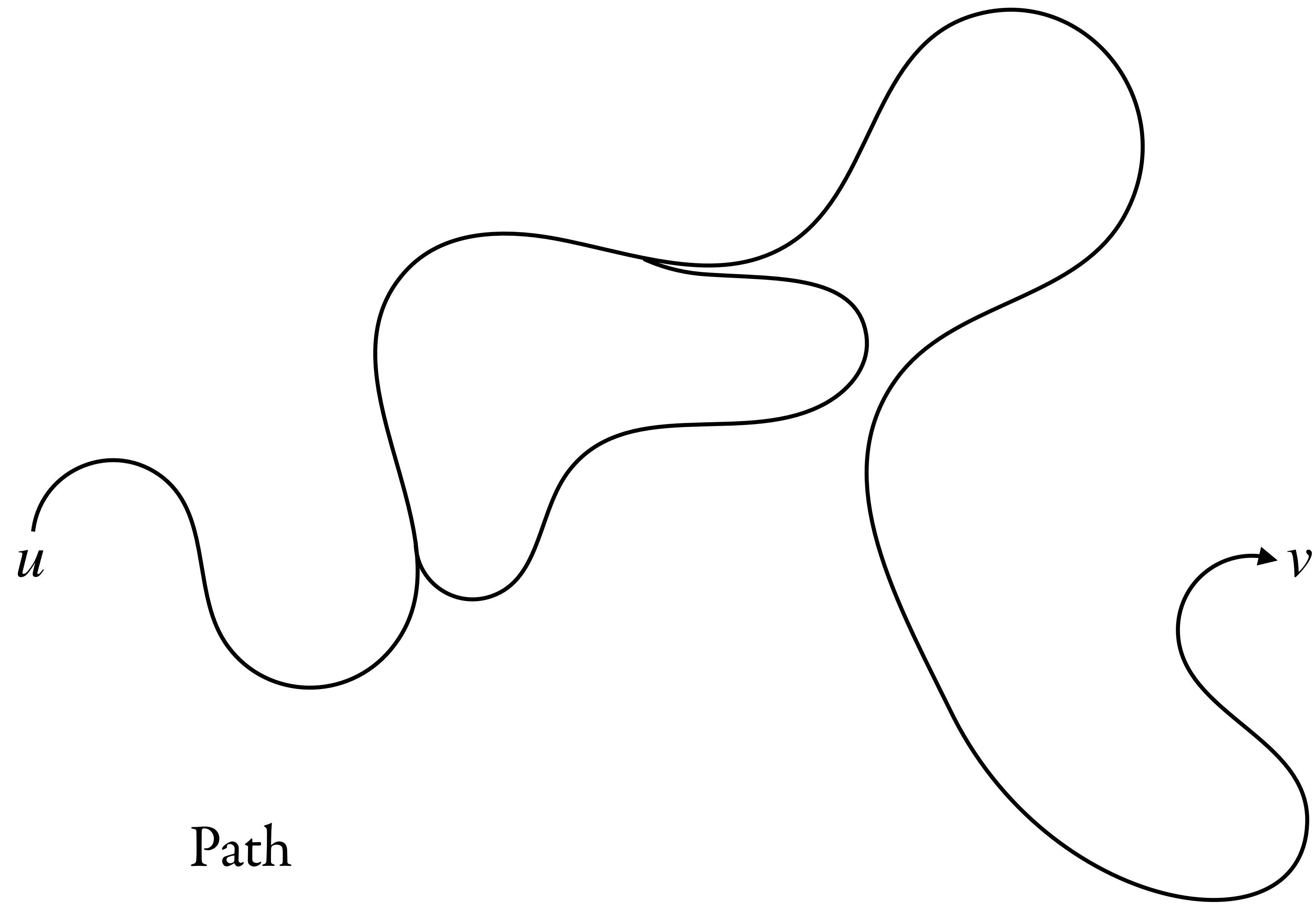

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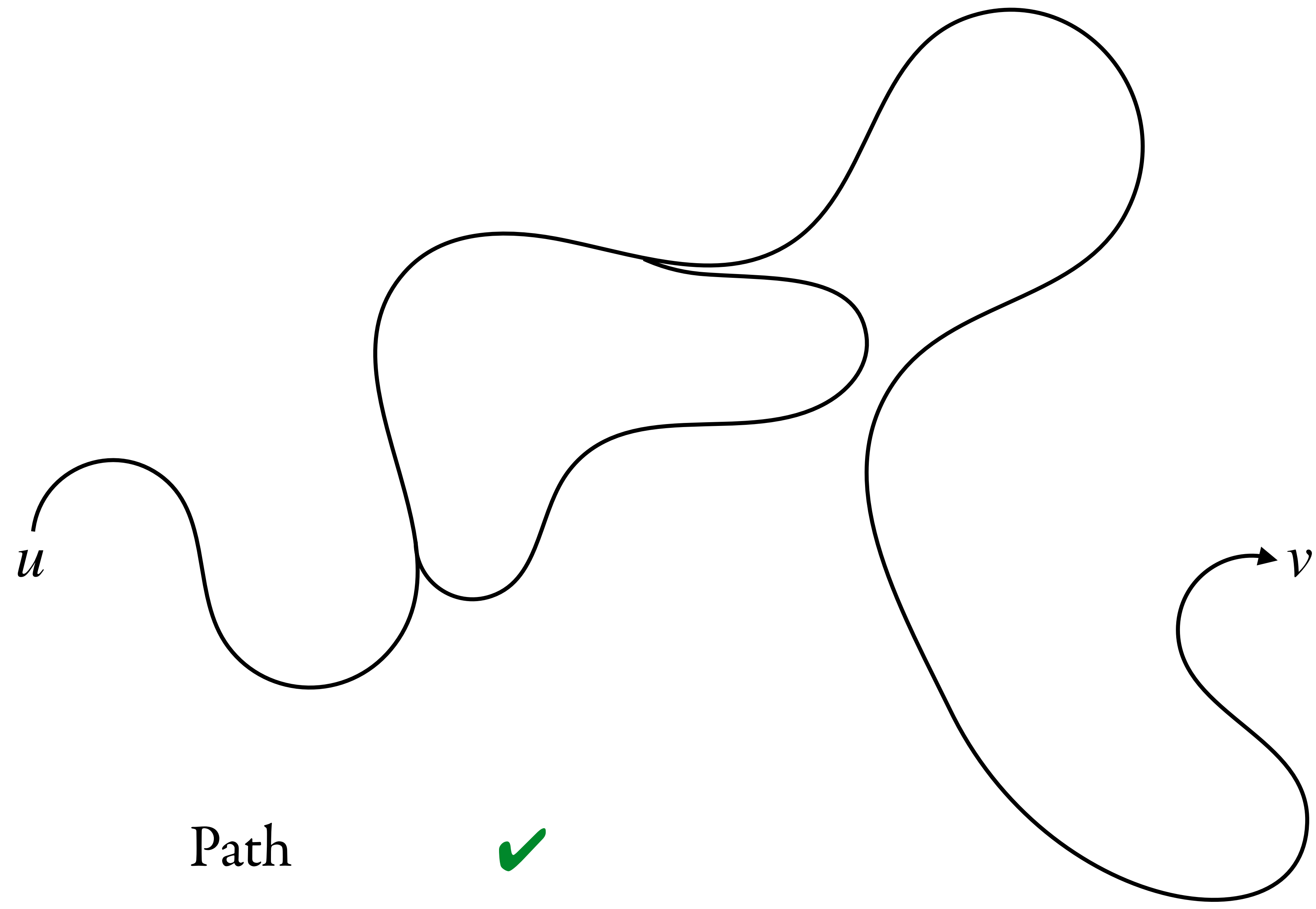
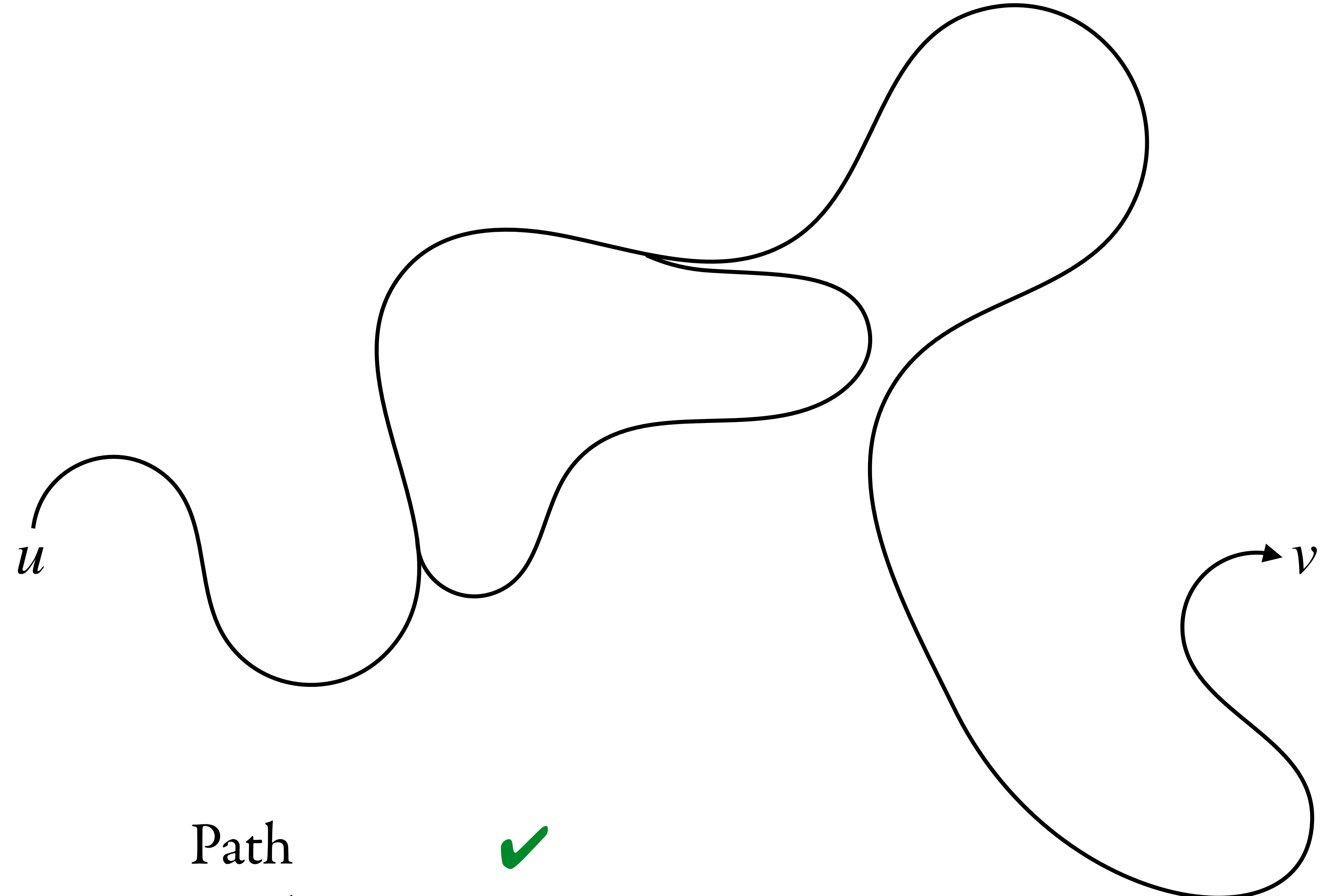


Path ✔

Simple Paths and Trails



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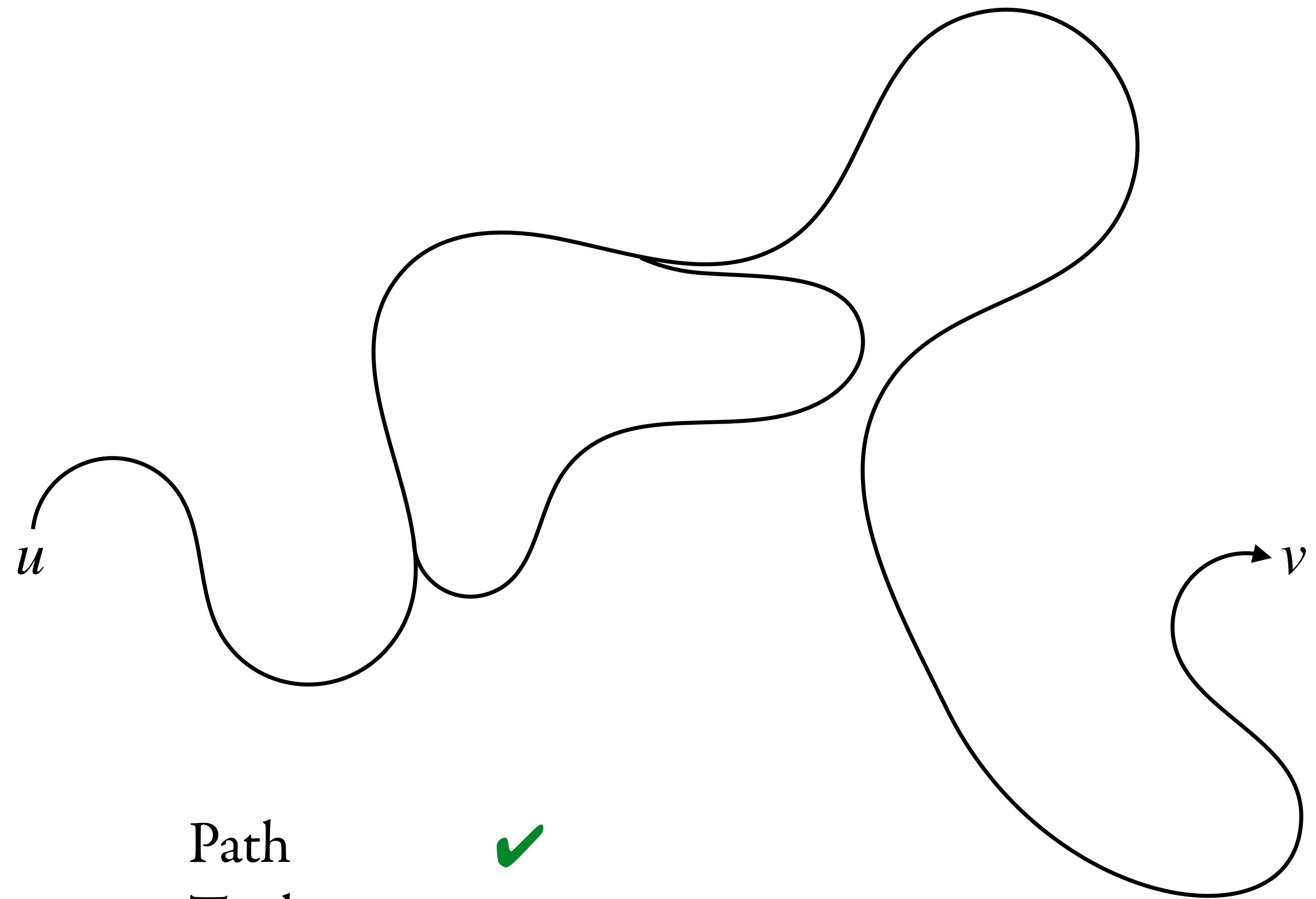

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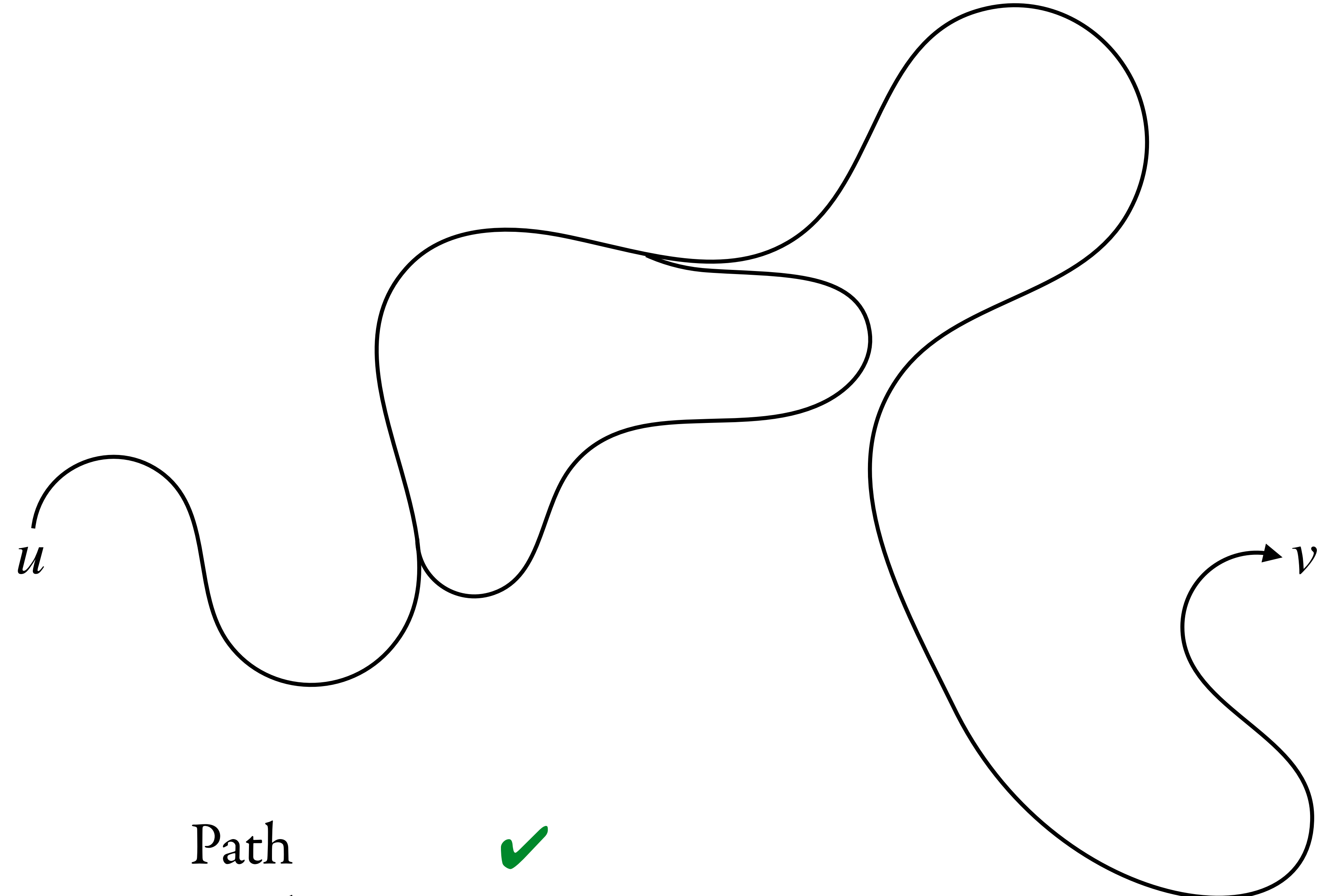

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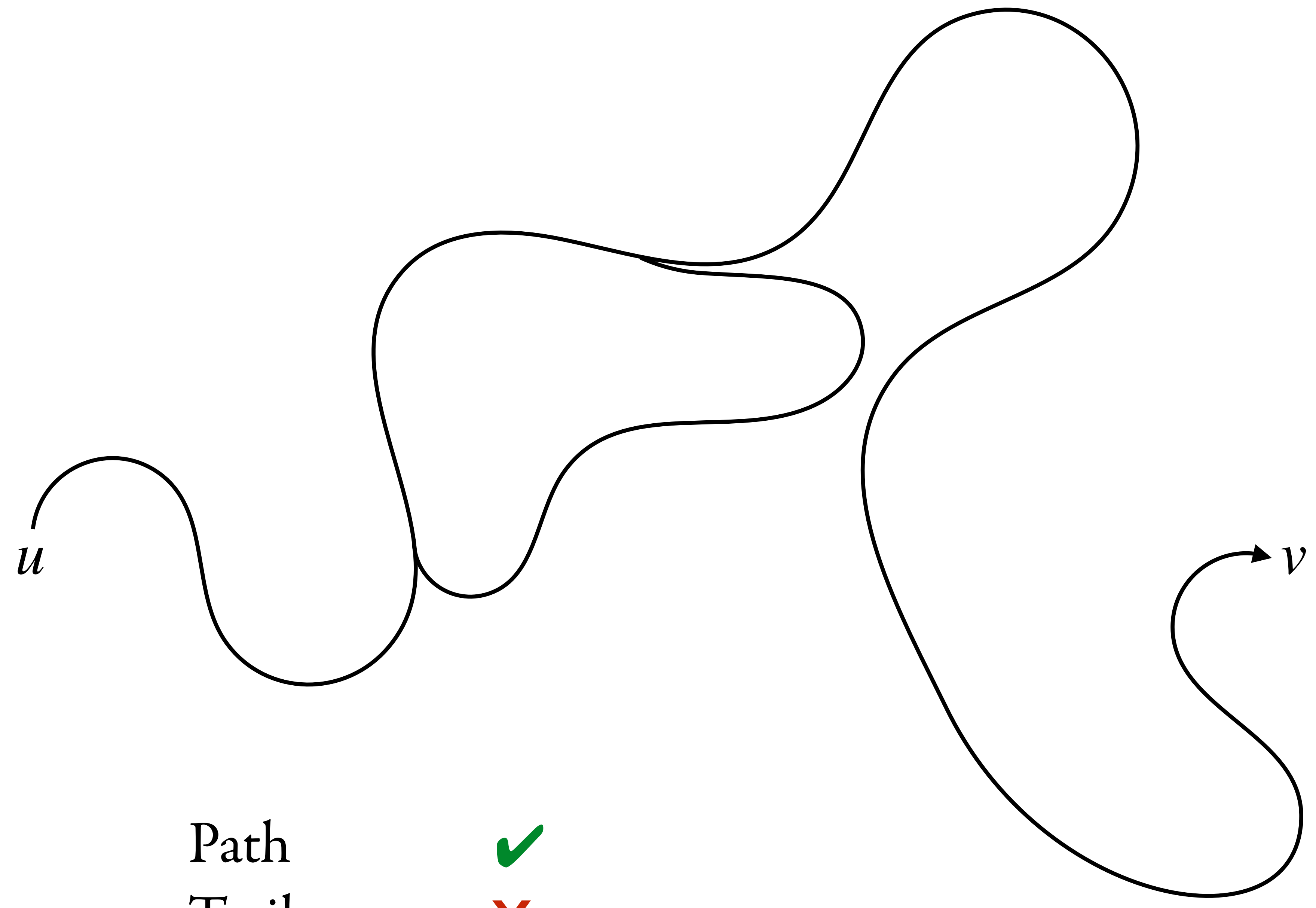

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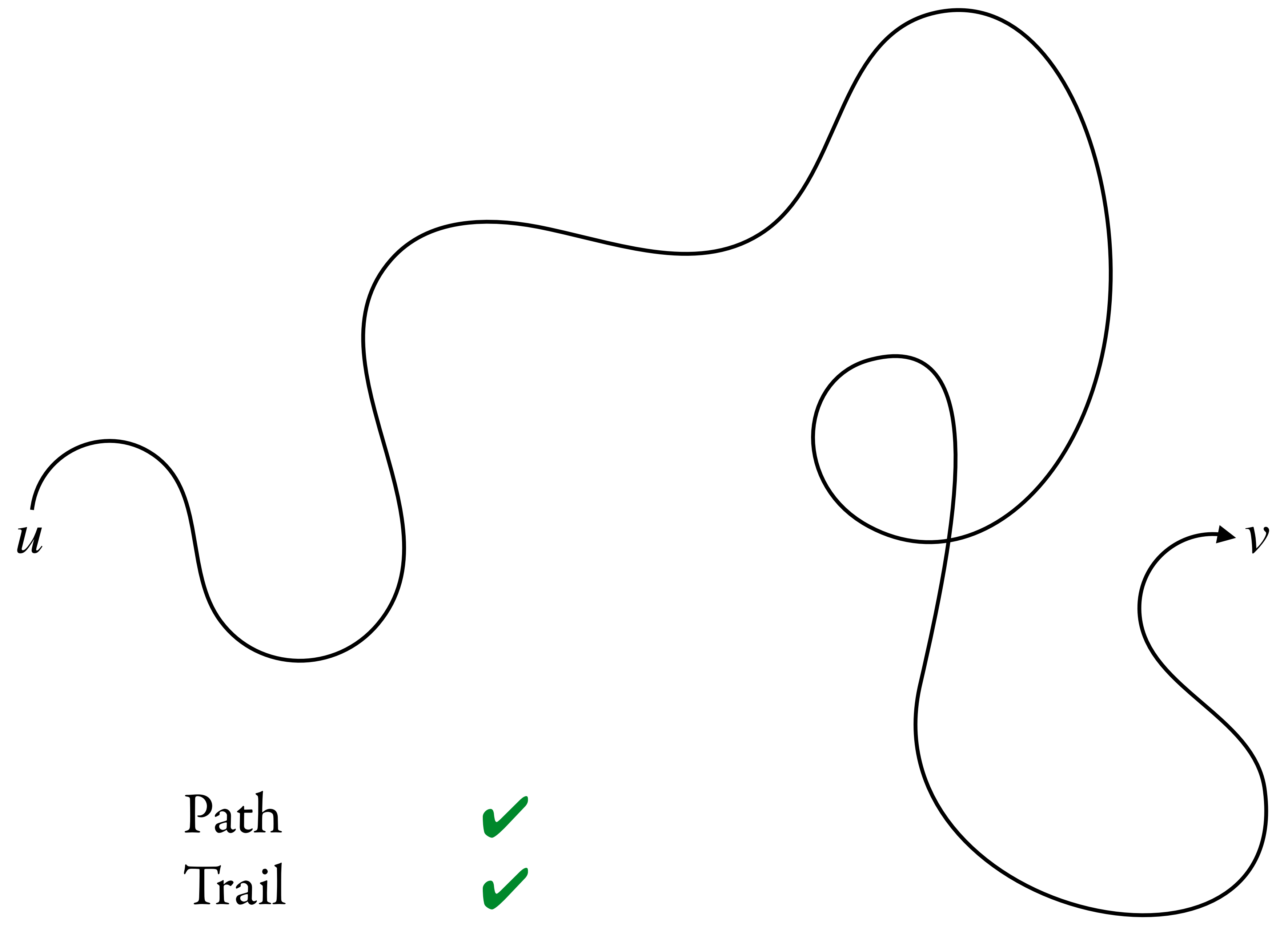

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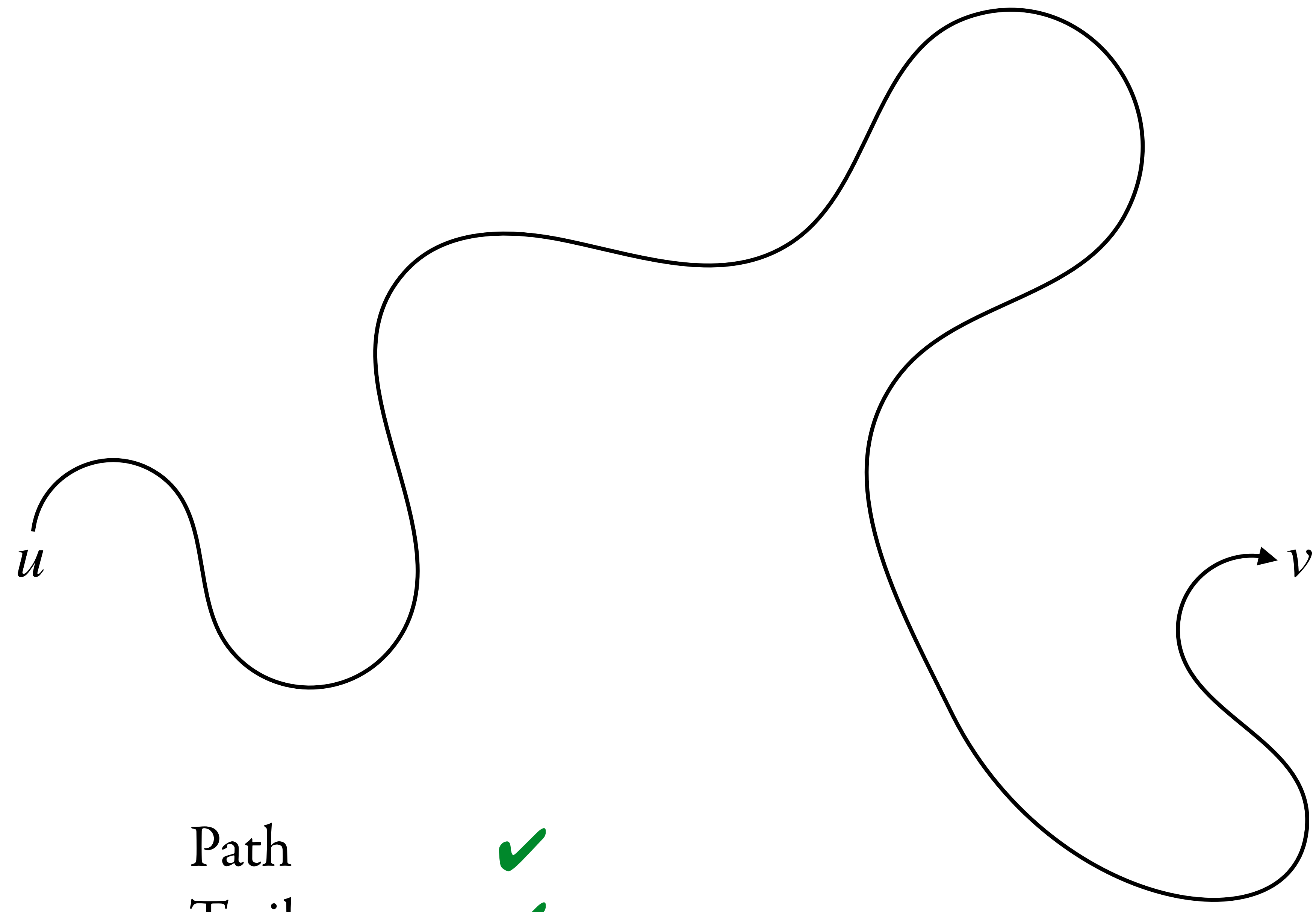

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Simple Paths and Trails

Definition (Simple path, trail)

Let $p = (v_0, a_1, v_1)\ (v_1, a_2, v_2)\ \ldots\ (v_{n-1}, a_n, v_n)$ be a path

Path p is a **simple path** if it is empty or

- v_0, v_n appear exactly once and
- every node in $\{v_1, \ldots, v_{n-1}\}$ appears exactly twice in p

Path p is a **trail** if it is empty or

- every edge (v_{i-1}, a_i, v_i) appears exactly once in p

Semantics of RPQs

Semantics of RPQs (simple path semantics)

Let $Q = (x \xrightarrow{r} y)$ be an RPQ and $G = (V, E)$ be a graph

The **answer of Q on G under simple path semantics** is

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Semantics of RPQs

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Let $Q = (x \xrightarrow{r} y)$ be an RPQ and $G = (V, E)$ be a graph

The **answer of Q on G under trail semantics** is

$$Q(G)_t = \{(u, v) \in V \times V \mid \text{there exists a trail } p \text{ from } u \text{ to } v \text{ in } G \text{ that matches } r\}$$

RPQ Semantics: Examples

Take $r = (aa)^*$

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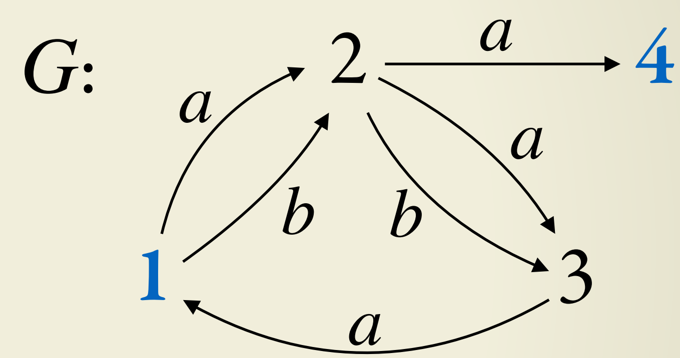

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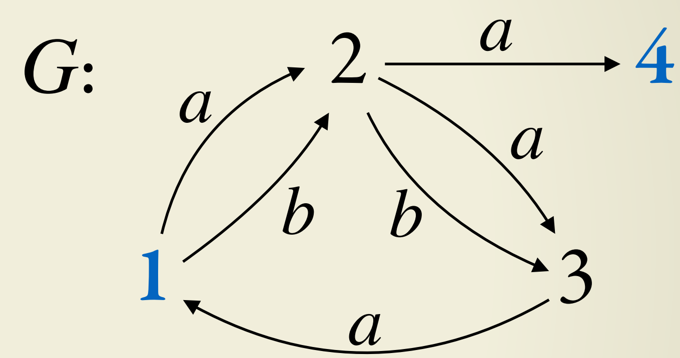

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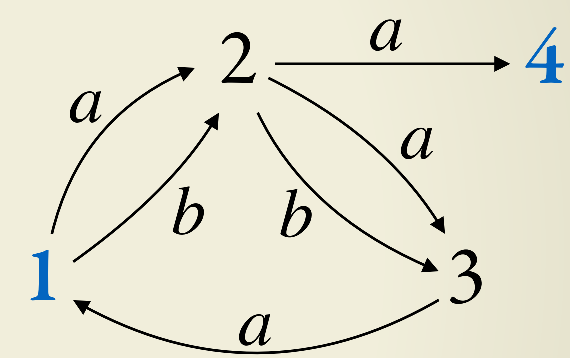

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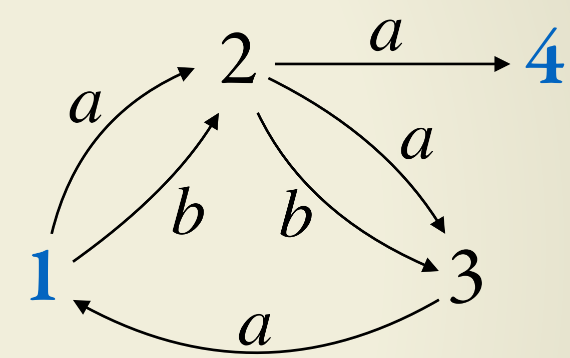

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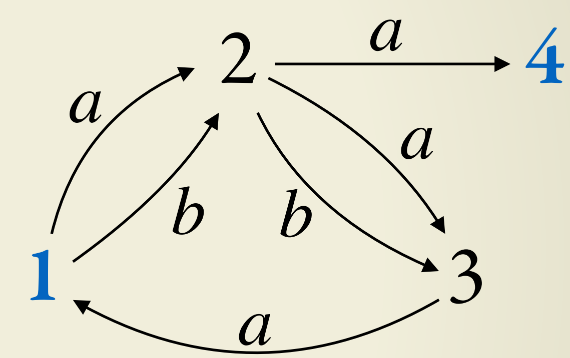

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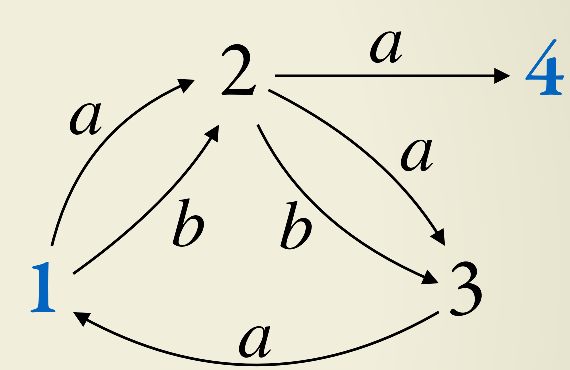

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Conjunctive Regular Path Queries

Definition (Conjunctive Regular Path Query)

A conjunctive regular path query (CRPQ) is an expression of the form

$$Q(\bar{x}) := \big((y_1 \xrightarrow{r_1} z_1) \wedge \cdots \wedge (y_n \xrightarrow{r_n} z_n)\big)$$

where

- $\bar{x}$ is a tuple of variables from $\{y_1, \ldots, y_n, z_1, \ldots, z_n\}$ and
- $(y_i \xrightarrow{r_i} z_i)$ is an RPQ over Σ for all $i \in [n]$

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Observation 1

Essentially a CQ where building blocks are RPQs

Observation 2

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Conjunctive Regular Path Queries

Semantics of CRPQs (every path semantics)

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The set of **answers of Q on G** (under **every path** semantics) is

$Q(G) = \{\, \mathrm{h}(\bar{x}) \mid \mathrm{h}$ is a homomorphism from vars(Q) to V

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Notation: $Q(G)_s$ for simple path semantics

$Q(G)_t$ for trail semantics

Conjunctive Queries (CQs)

Query Evaluation

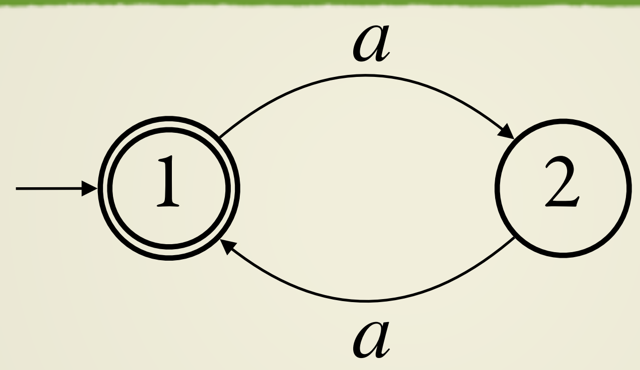
Regular Path Queries (RPQs)

Conjunctive Regular Path Queries (CRPQs)

Notation and Basic Principles

If $n \in \mathbb{N}$, we use $[n]$ to denote the set $\{1, \ldots, n\}$

Finite Automata



We denote a **nondeterministic finite automaton (NFA)** as

$$N = (S, A, \delta, I, F)$$

where

- S is the finite set of states
- A is the finite alphabet
- $\delta \subseteq S \times A \times S$ is the transition relation
- $I \subseteq S$ is the set of initial states
- $F \subseteq S$ is the set of accepting (or "final") states

The **language of N** is denoted $L(N)$

In the example:

$S = \{1,2\}$

$A = \{a\}$

$\delta = \{(1,a,2), (2,a,1)\}$

$I = \{1\}$

$F = \{1\}$

Evaluation Problems

RPQ Evaluation (every path semantics)

Input: Graph database G, pair (u, v) of nodes
regular path query Q

Question: Is $(u, v) \in Q(G)$?

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The decision problems for simple path and trail semantics are defined analogously

RPQs, Every Path Semantics

Theorem

RPQ Evaluation under every path semantics is in PTIME

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Construct a product $G \times N$, treating u as "initial state" in G

(This is similar to a product between automata)

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Construct a product $G \times N$, treating u as "initial state" in G

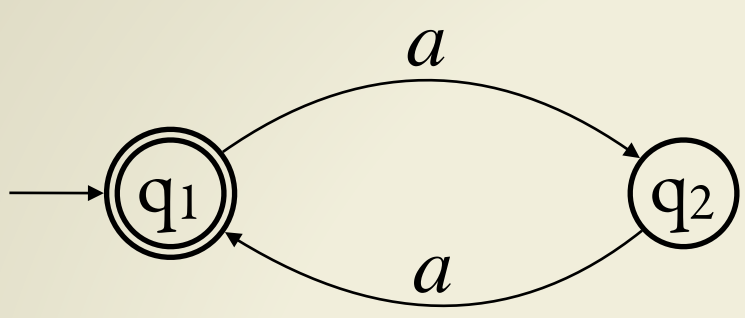
(This is similar to a product between automata)

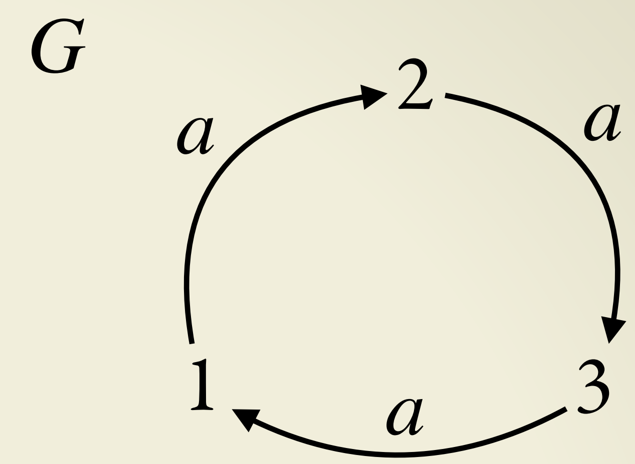
Accept iff there is a path from (i,u) to (f,v) in $G \times N$, for some $i \in I$ and $f \in F$

Example

RPQ Evaluation under Every Path Semantics

Consider the RPQ $r = (aa)^*$





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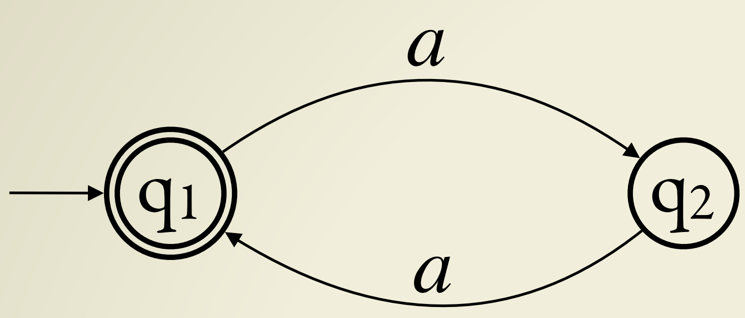

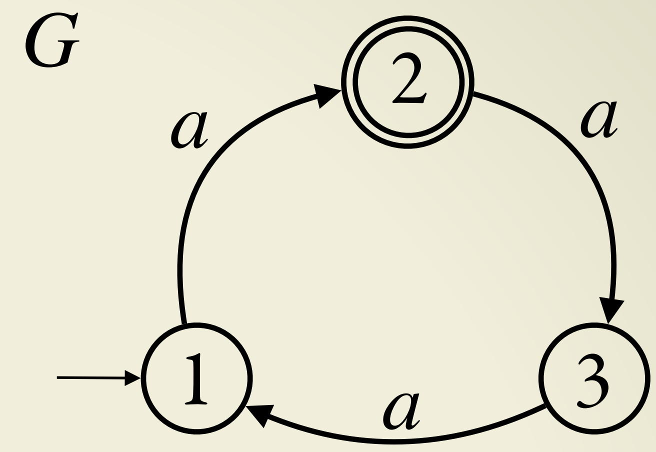

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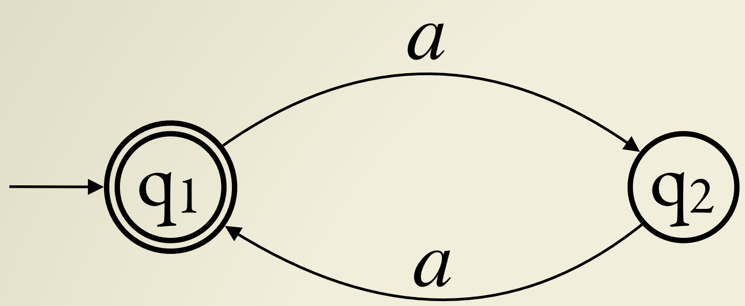

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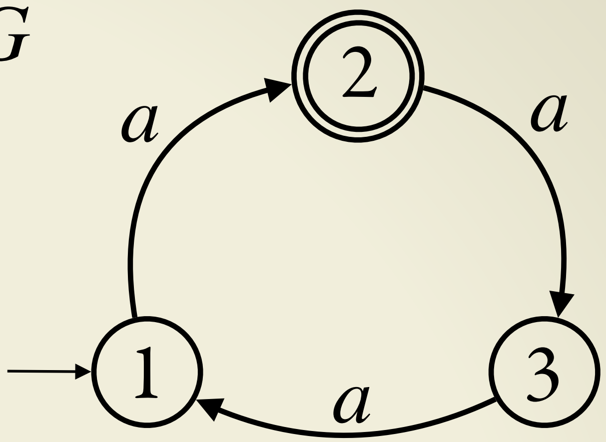

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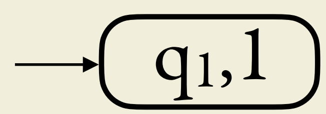

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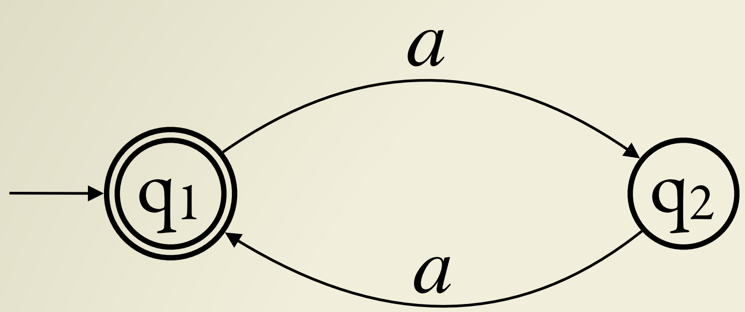

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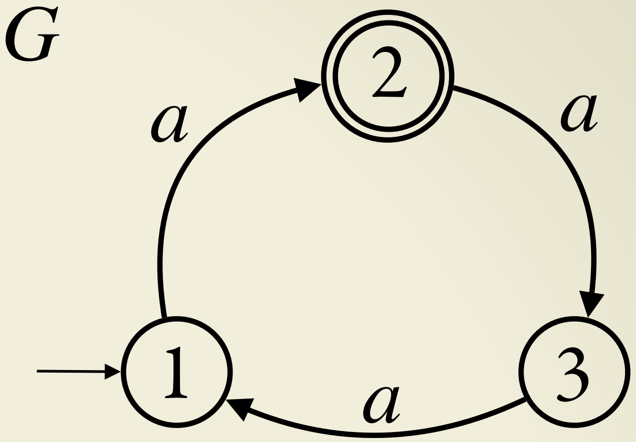

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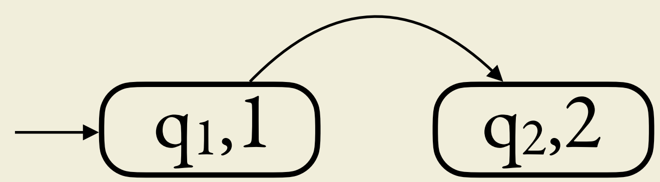

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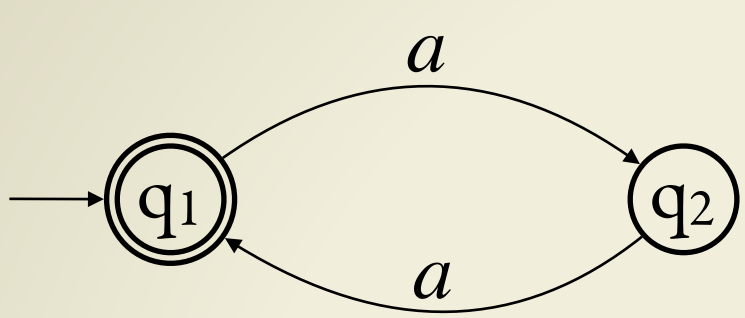

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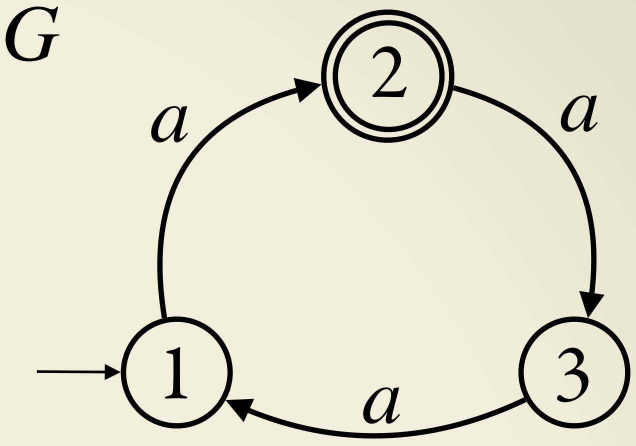

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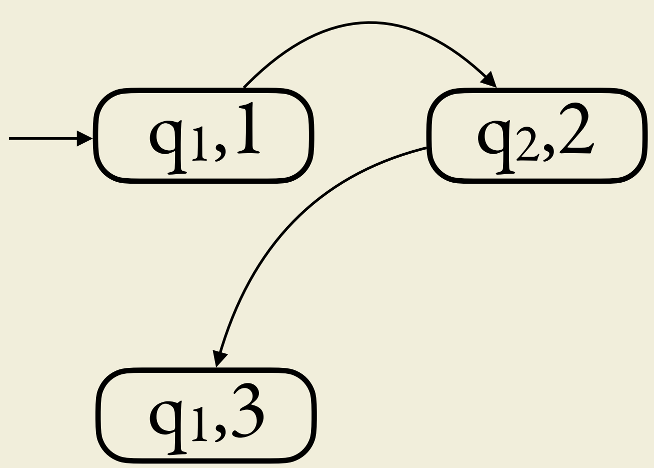

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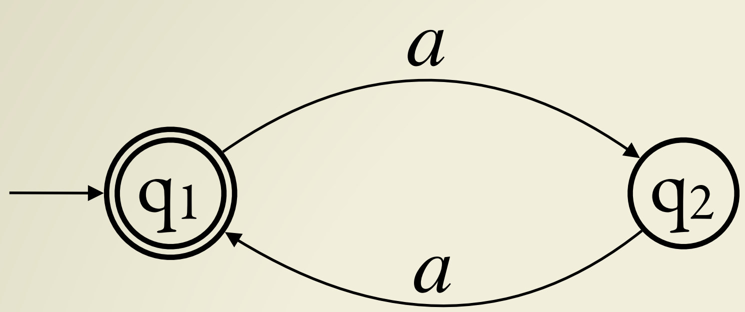

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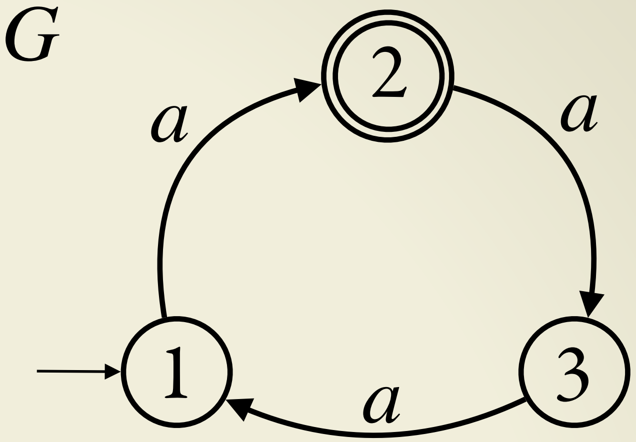

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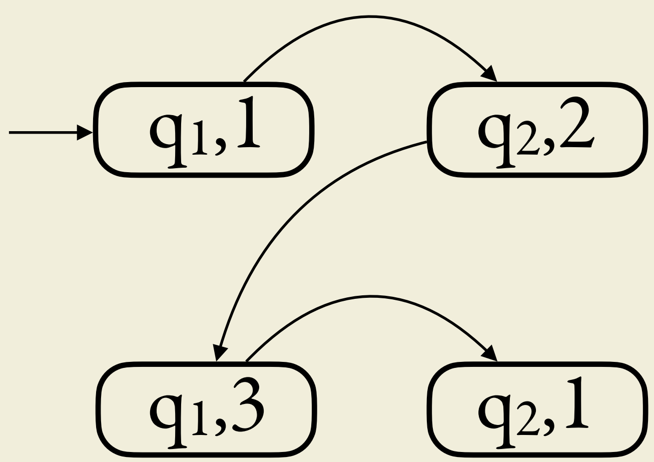

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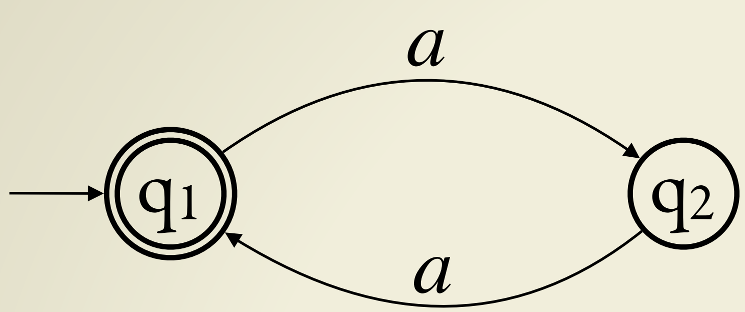
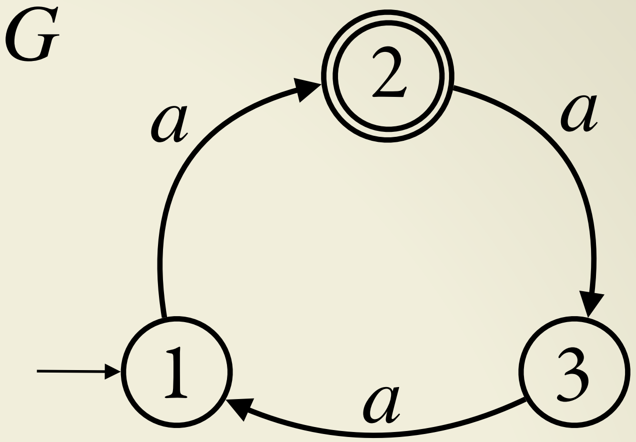
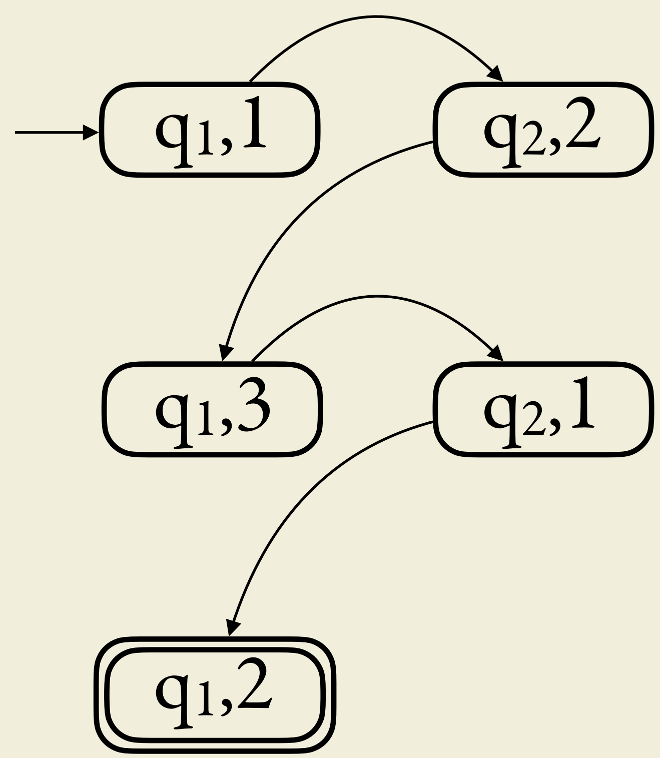


G



Is (1,2) in $r(G)$?



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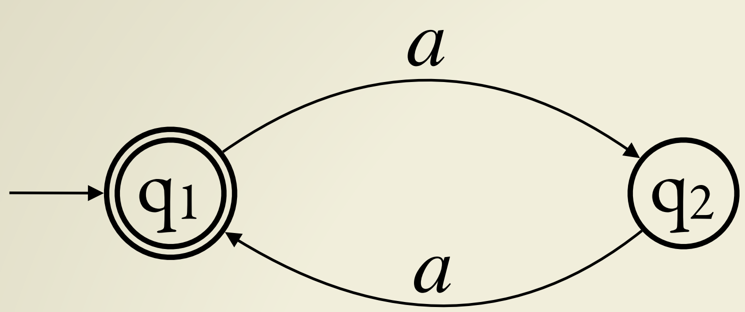

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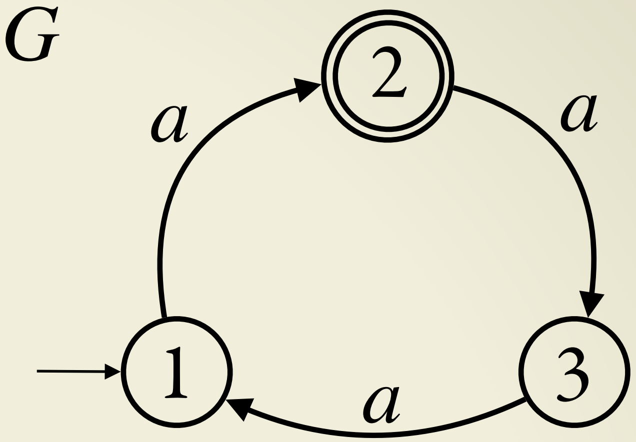

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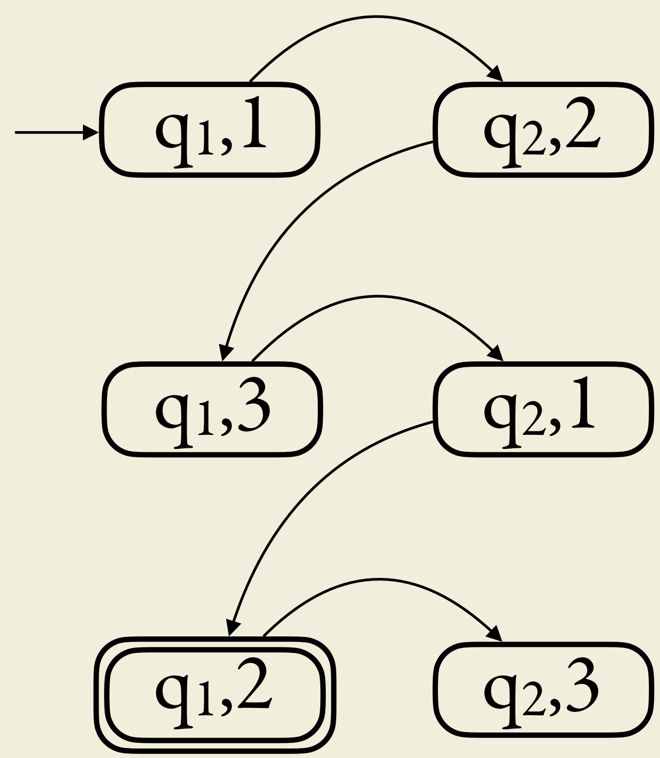

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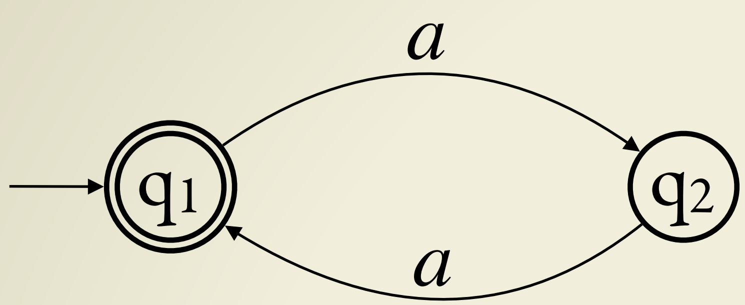

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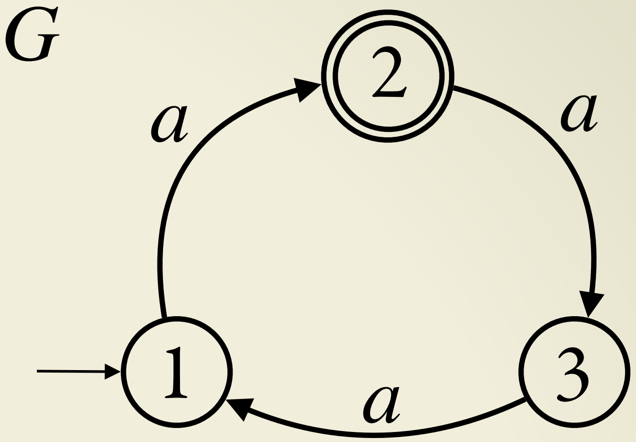

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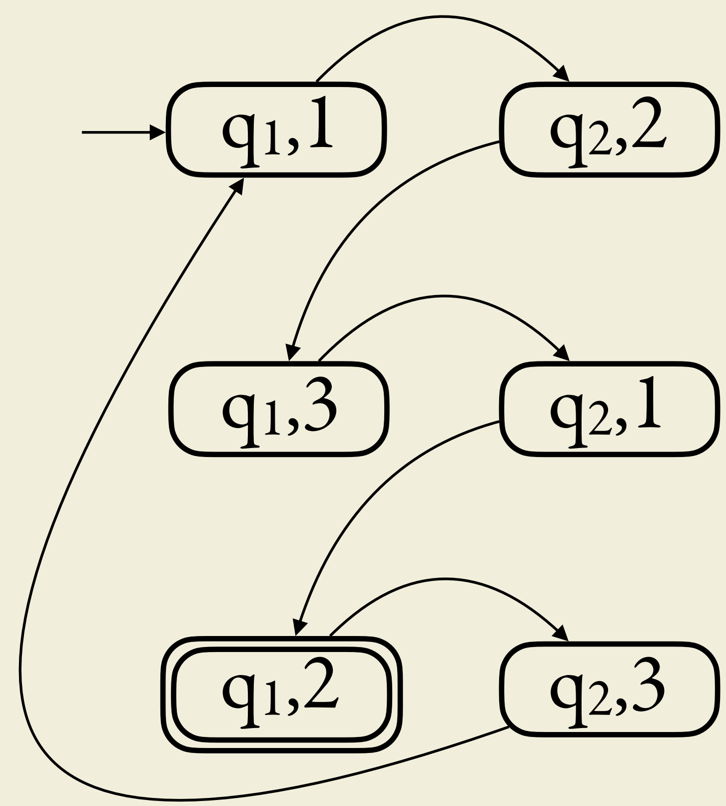

RPQs, Simple Path Semantics

Theorem

RPQ Evaluation under simple path semantics is NP-complete

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Proof (sketch)

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Let G_a be the graph constructed before

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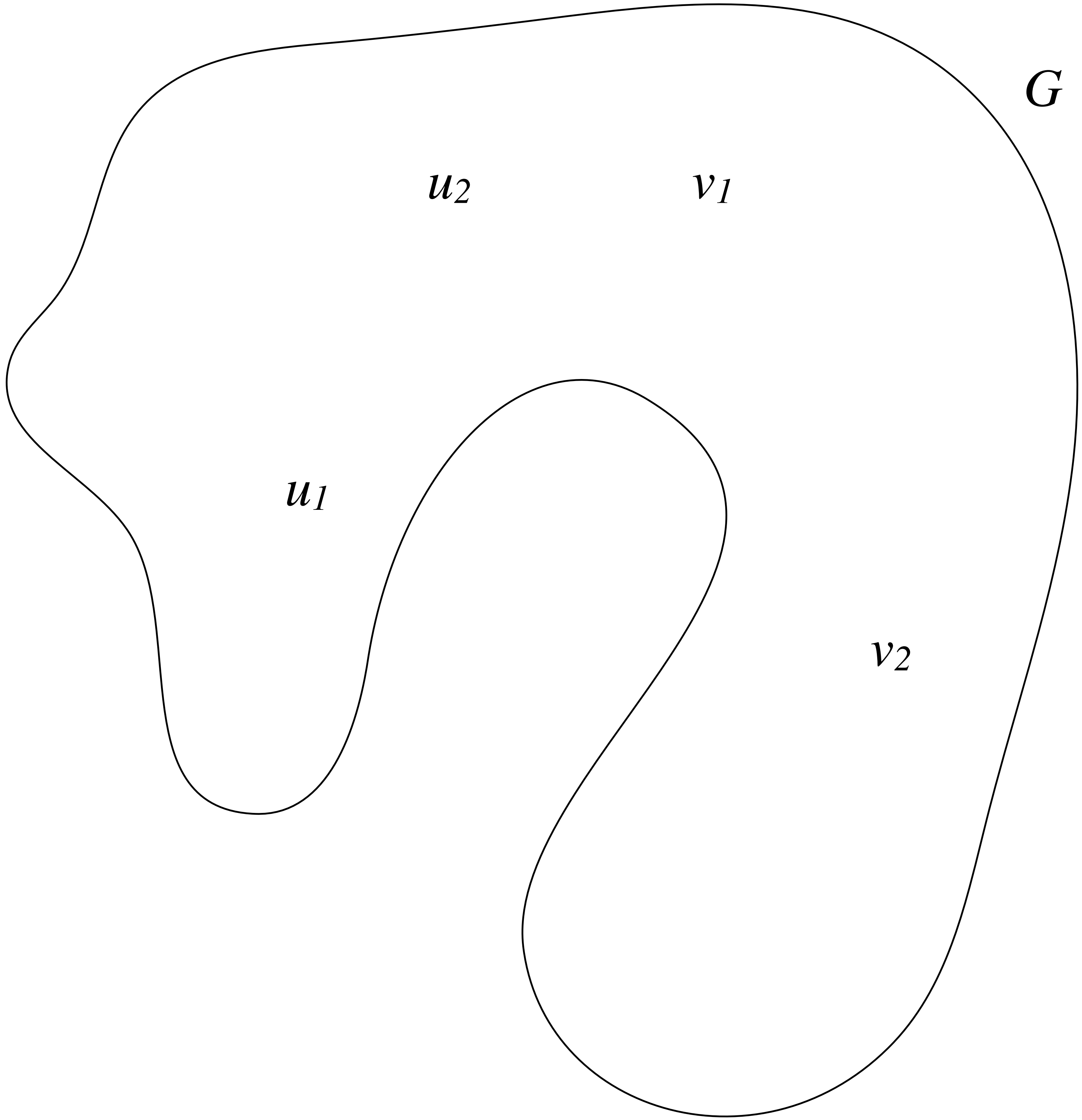

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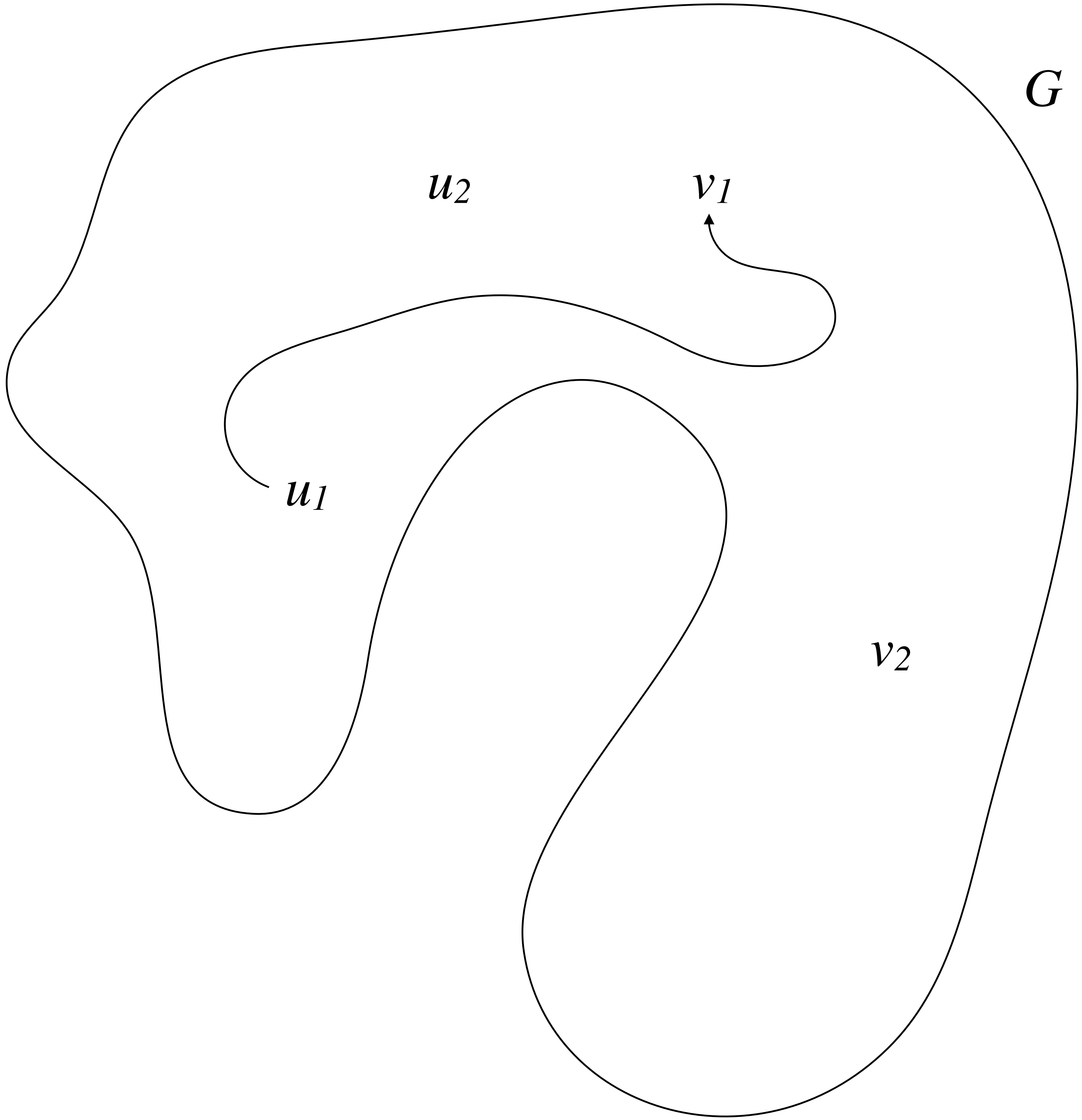

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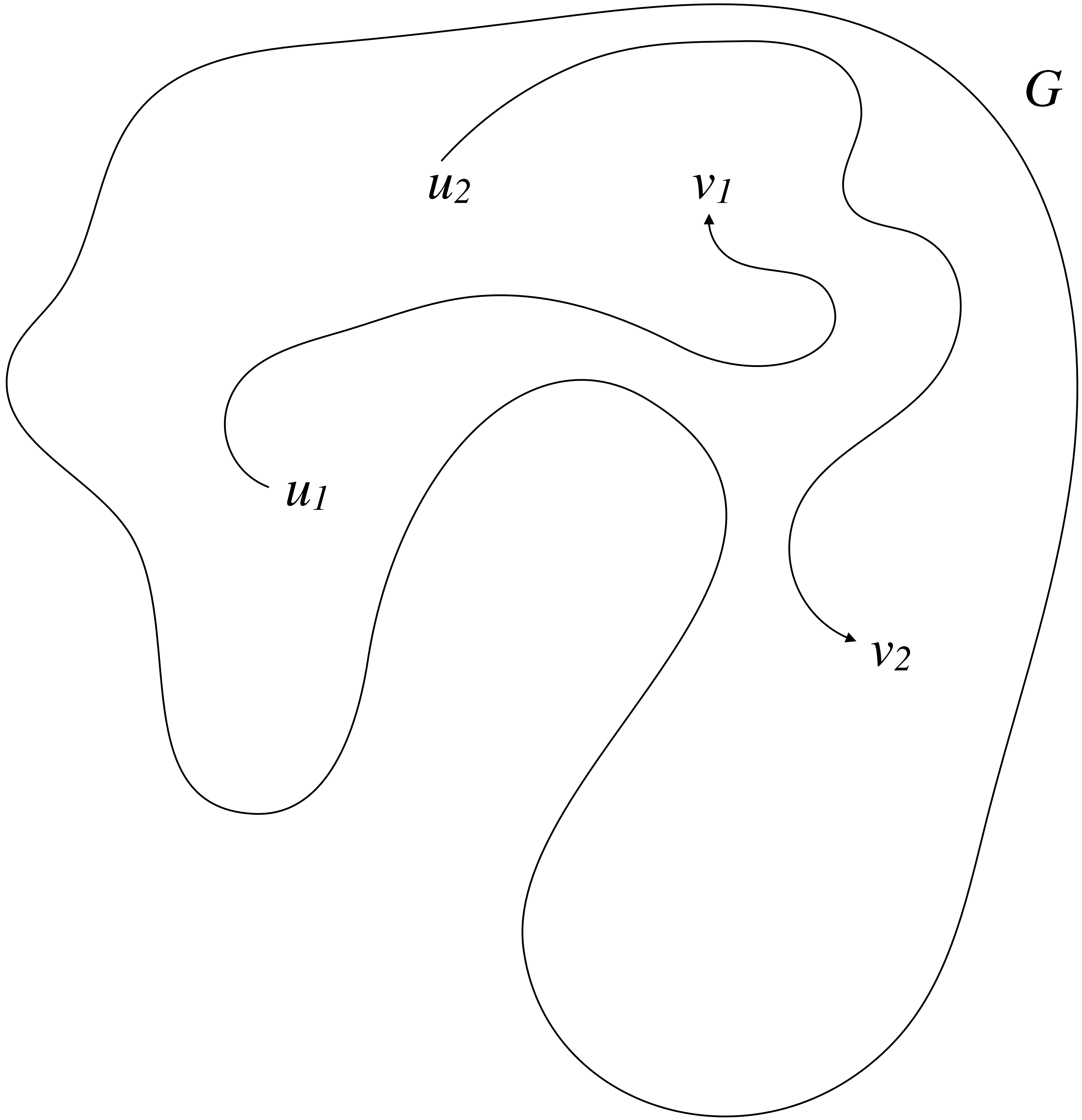

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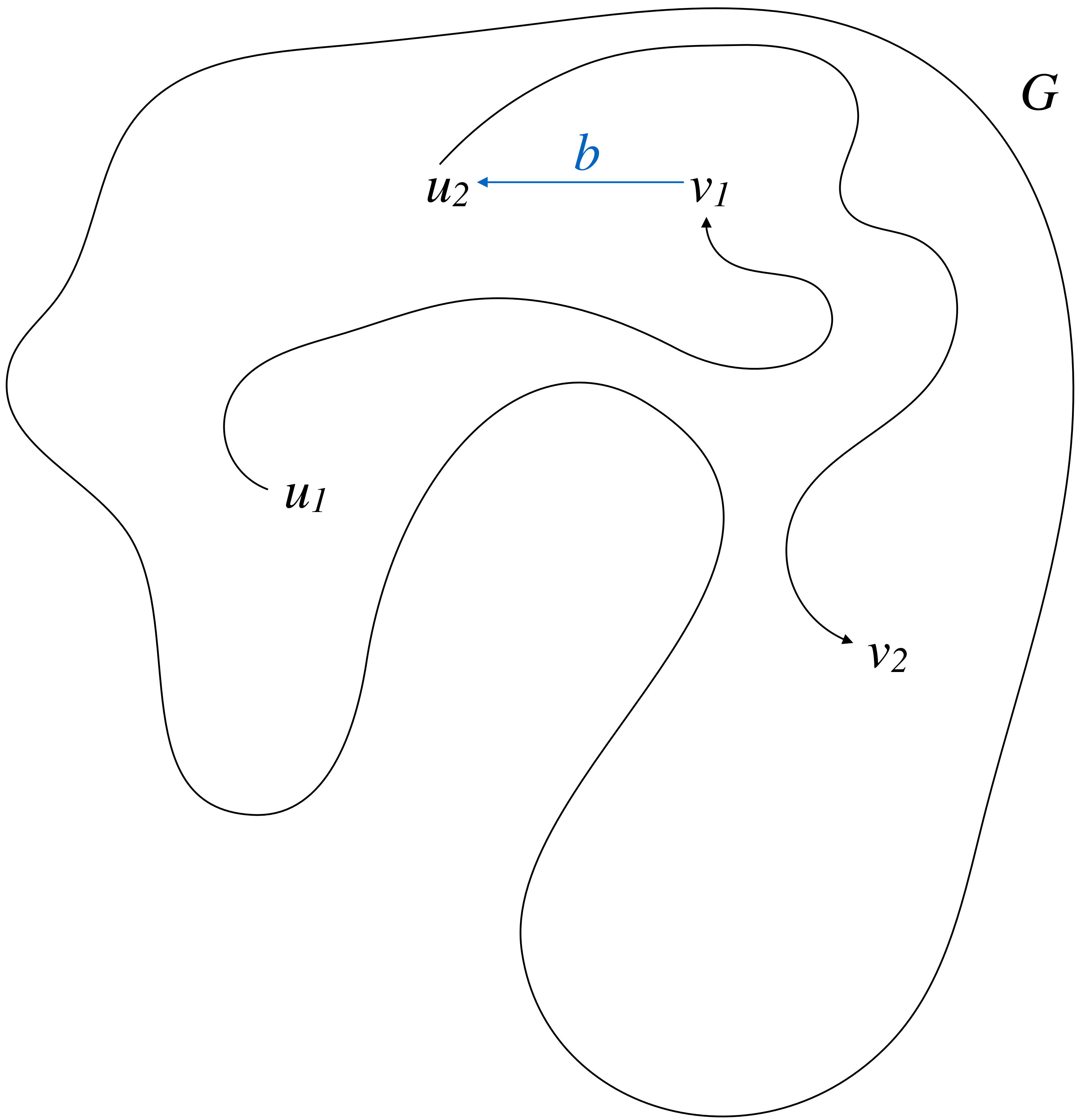

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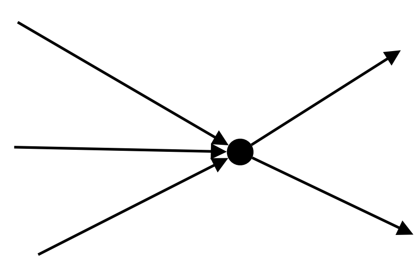

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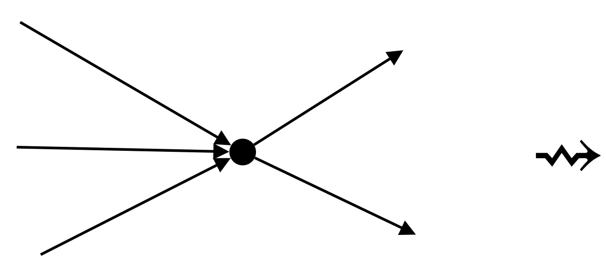

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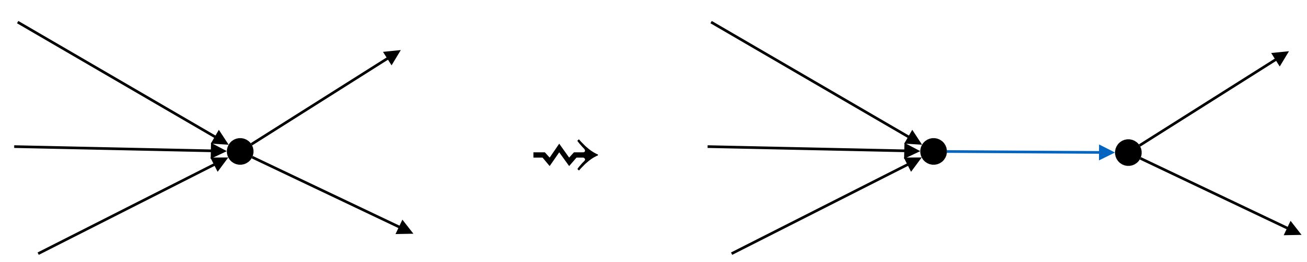

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Proof (sketch - same reduction as before)

Let G_b be obtained from G_a by adding the edge (v_1, b, u_2)

Then G has edge-disjoint paths p_1 and p_2, from u_1 to v_1 and from u_2 to v_2 iff

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A similar proof.

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a relation R_i containing the pairs $r_i(G)$

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Let C be a class of CRPQs

Let C_{Rel} be the class of (relational) CQs, defined as $C_{Rel} = \{Q_R \mid Q \in C\}$

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So, by the results on tree-shaped conjunctive queries,
evaluation on tree-shaped CRPQs is also tractable

CRPQs, Simple Path / Trail Semantics

Theorem

CRPQ Evaluation is NP-complete under simple path and under trail semantics

CRPQs, Simple Path / Trail Semantics

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Proof (sketch)

Lower bound: already holds for RPQs

Upper bound: simple guess-and-check algorithm

Overview

	RPQs	CRPQs
every path	PTIME	NP-complete
simple path	NP-complete	NP-complete
trail	NP-complete	NP-complete

Basic Containment Problems

RPQ Containment

Input: RPQs Q_1 and Q_2

Question: Is $Q_1(G) \subseteq Q_2(G)$ for every graph G?

CRPQ Containment

Input: CRPQs Q_1 and Q_2

Question: Is $Q_1(G) \subseteq Q_2(G)$ for every graph G?

The problems for simple path and trail semantics are analogous

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Testing $L(r_1) \subseteq L(r_2)$ for two given regular expressions r_1 and r_2 is PSPACE-complete

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The same proof works for simple path and trail semantics

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Data Values

Queries With Data Value Comparisons

Until now, we never compared labels with each other

Example:

- Return pairs of people with the same last name

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Such queries are usually considered on a different data model

(data words, data trees, data graphs)

but since we chose Σ infinite, the main argument also works here

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Consider the query L_{eq}, matching all paths that contain two equal values

Language L_{eq} is the most basic one imaginable that compares data values.
Hence regular expressions should avoid complementation.

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expresses L_{eq}: bind x, see if it occurs elsewhere ($[=x]$)

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Consider the query L_{eq}, matching all paths that contain two equal values

Let $\overline{L_{eq}}$ be its complement,

matching all paths containing pairwise different values

Theorem

Evaluation of $\overline{L_{eq}}$ on graph databases is NP-complete

Language L_{eq} is the most basic one imaginable that compares data values.
Hence regular expressions should avoid complementation.

Regular expressions with binding:

$$\Sigma^* \cdot \downarrow x \cdot \Sigma^+[=x] \cdot \Sigma^*$$

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also expresses L_{eq}: guesses where equal values occur